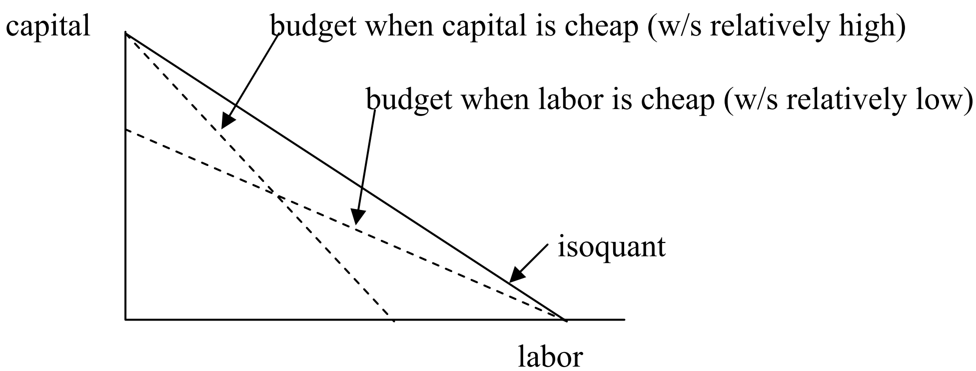
Econ 463 R. Butler 2005 Lecture 7

More Than One Input: Substitution and Scale Effects

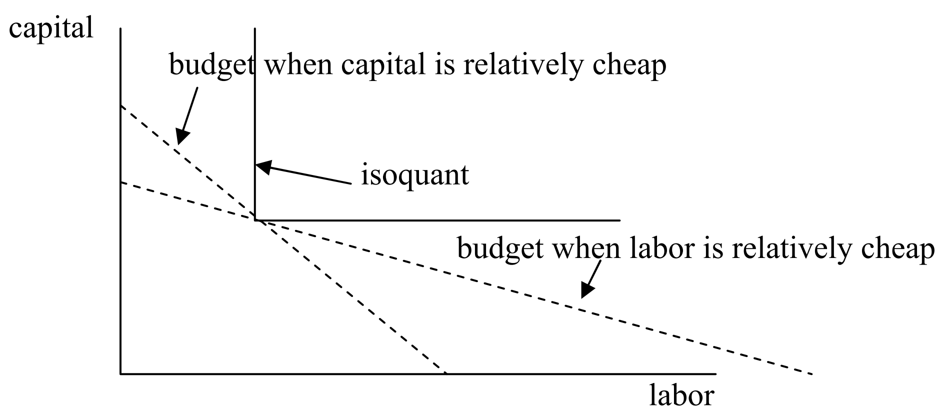
I. production function sculptures: the perfect substitutes, no substitution, and some substitution cases

isoquant=combinations of inputs that produce the same level of output

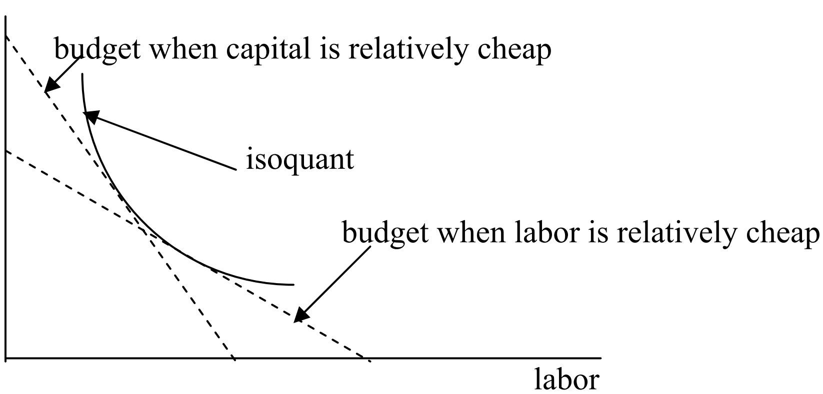
A. inputs are perfect substitutes case (dashed lines are budget constraints)



B. no substitution between inputs case (dashed lines are budget constraints)



C. intermediate case: some, but not perfect substitution



The demand for labor is a derived demand, depending on

w,s, and Y (input prices and the level of output—this is the usual formulation, known as a conditional demand function and is based on the cost function)

or

w,s, and P (input prices and output price—based on the profit function, which we don't discuss here)

or

w,s, and C (input prices and total costs, rarely used)

Let's see how to derive the demand for labor for no substitution and some subsitution production cases, in the process formally defining homogeneous, homothetic, and non-homethetic production functions.

II. **Long-run demand for labor: no factor substitution case and CRS:**

$Y = \min(b_L L, b_K K)$ case

where prod. fn (to each firm): $Y = \min(b_L L, b_K K)$ where min(a,b) means "take the smaller of a or b"

example with $b_L=3$ and $b_K = 2$

Y	K	L	Y	K	L	Y	K	L
12	6	4	12	6	100	2	1	4
12	6	6	12	7	4	9	6	3
12	100	4	2	1	4	3	6	1

also assume that:

two homogeneous inputs: L=labor, K=capital

all firms have the same production function with CRS

Note: b_L, b_K are technology parameters equal to some positive real number (so as b_L increases, more output is possible with the same amount of labor input).

Proof that the $Y=\min(b_L L, b_K K)$ is CRS:

suppose that $b_L L$ is the smaller of the two terms or that the two terms are of the same size, then when we double each term:

$\min(2b_L L, 2b_K K) = 2\, b_L L = 2\, Y$. (since $b_L L = Y$ by the definition of the function)

suppose that $b_K K$ is smaller of the two terms, then when we double each term:

$\min(2\, b_L L, 2b_K K) = 2\, b_K K = 2\, Y$. (since $b_K K = Y$ by the definition of the function)

Either way, doubling the inputs doubles the output. QED.

While you can't differentiate the isoquant at the kink point, you can find those conditions that minimize costs:

$$b_L L = b_K K = Y$$

Why do these two relationships have to hold in order to minimize costs? Substitute these "first order conditions" (rewritten as $L = Y/b_L$ and $K = Y/b_K$) into the cost constraint to get the cost function (costs as a function of output and input prices)

$$C = w*Y/b_L + s*Y/b_K = Y(w/b_L + s/b_K)$$

For the price, the supply price will be the marginal cost, which from the cost function will be

$$\frac{\partial C}{\partial Y} = \frac{w}{b_L} + \frac{s}{b_K} = P^s$$

The demand for the output Y depends on the price and anything that shifts the demand curve (call those shift factors "D"). In equilibrium, the supply price equals the demand price; so the equilibrium price will depend on the costs of inputs, in particular

Total market Demand for $Y = h(P^s, D) = h(w/b_L + s/b_K, D)$. The first order condition

$$L = Y/b_L$$

shows that the demand for labor *does not* depend on input prices directly when you have a Leontiff-type production function, but only on the level of output and the technology coefficient b_K. That is, there is no substitution effect in the model. But the demand for labor indirectly depends on input prices because the cost of labor and capital determine the marginal costs of production, and hence the equilibrium price level. Since the quantity of labor demanded depends on the product price, input prices will—through a scale effect—affect the quantity of labor demanded. Since Y is the output of the firm, and L is the labor services employed by the firm, then if n=number of firms, the total labor demanded will be n*L when the total output produced is n*Y. So the aggregate demand condition is

$$n*L = n*Y/b_L$$

Letting L^a be n*L, and Y^a be n*Y, we can write all this mathematically, substitute the demand function into the first order condition to get

$$L^a = Y^a/b_L = h(w/b_L + s/b_K, D)/b_L$$

This labor demand equation for this first production function has the following characteristics:

*input prices matter through a scale effect (input prices determined the marginal cost of the product, and hence the equilibrium price and quantity of output—and hence, of labor demanded), even if there is no substitution effect

*the effect of technological change on the demand for labor is ambiguous: it takes less labor to produce the same output Y, so demand will fall on this score BUT it also lowers the cost of production, and hence lowers the equilibrium product price. So quantity demanded of Y increases, rising labor demand. The net effect is indeterminate without additional assumptions.

*any factor D that shifts the demand for the product, will also affect the demand for labor

III. **Long-run demand for labor: no substitution and Homogeneous production:**

Now the production function is written as

$Y = [\min(b_L L, b_K K)]^{\pi}$ where π =returns to scale parameter, where:

π=1 CRS π<1 DRS π>1 IRS

Since $Y^{1/\pi}$ =min(b_LL,b_KK), the first order conditions for cost minimization is now

$Y^{1/\pi} = b_L L$, and $Y^{1/\pi} = b_K K$.

Substituting these into the cost constraint, we derive the cost function for a homogeneous production function as

$C = Y^{1/\pi}(w/b_L + s/b_K)$

Hence maraginal costs are given as $C' = \frac{\partial C}{\partial Y} = \frac{1}{\pi} Y^{1/\pi - 1}(\frac{w}{b_L} + \frac{s}{b_K})$,

and average costs are given as

$\overline{C} = \frac{C}{Y} = Y^{1/\pi - 1}(\frac{w}{b_L} + \frac{s}{b_K})$

The *marginal/average principle:* If the marginal (cost, revenue) is greater than the average (cost, revenue), then the average (cost, revenue) must be rising. If the marginal is less than the average, then the average must be falling. Why?

If $\pi > 1$ (IRS), then $C' < \overline{C}$, average costs falls and since P=C', then for each product sold, the revenue (the price) will be less than the average cost of producing the product so the competitive firm will be losing money. As we discussed in lecture 3, IRS is not compatiable with competitive pricing.

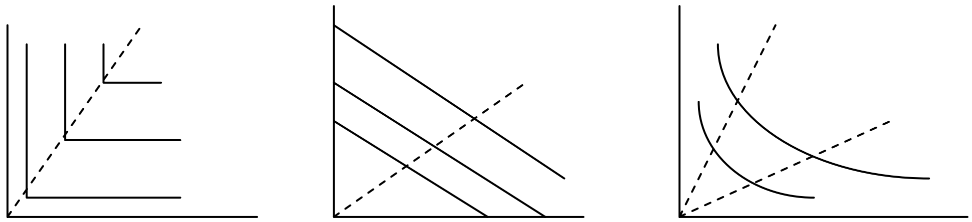
If $\pi = 1$ (CRS), then $C' = \overline{C}$, then product price will equal the average cost (and marginal cost), and there will be zero profits (the factors of production completely exhaust the revenue).

If $\pi < 1$ (DRS), then $C' > \overline{C}$, so average costs rise and since the price equals C', which is above $\overline{C}$, the firm makes profits. DRS and CRS are compatible with competitive pricing.

Long run equilibrium is such that no new firms have an incentive to enter, and all the producing firms will continue to produce. Among the homogeneous production functions, only CRS is compatible with long run equilibrium. Why?

IV. **Long-run demand for labor: no substitution and Homothetic production:**

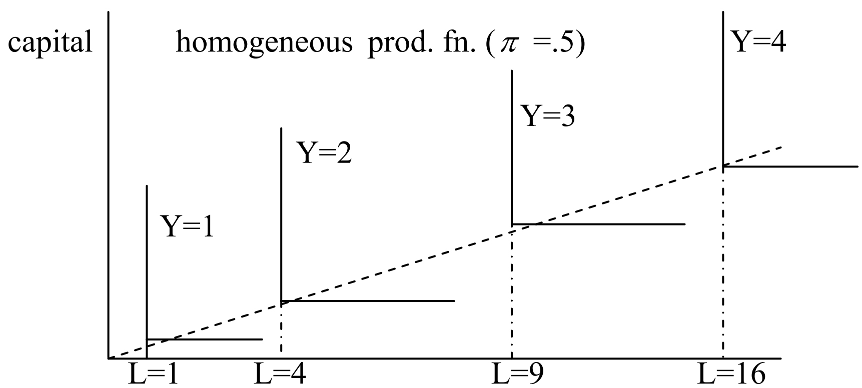
Homothetic functions have isoquants that are all radial expansions (from the origin) of each other; they all have the same shape; and for any given K/L ratio, the marginal rate of substitution between K and L (given as the slope of the isoquant along the given K/L ratio) is the same. Examples of homothetic functions include:



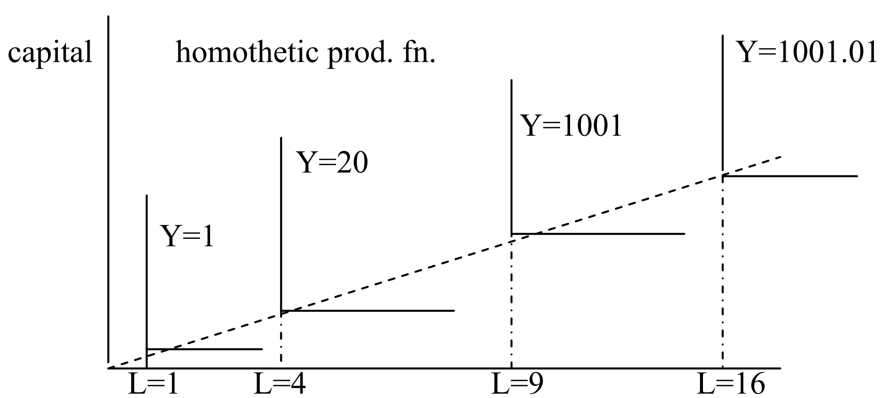
Homogeneous production functions are special cases of homothetic functions in which the isoquants have a "smooth" labeling (in our case, the labeling follows an power function). Homothetic functions are monotonic transformations of homogeneous functions:

$$g(Y) = \min(b_L L, b_K K)$$

where g(0)=0 and g'(Y)>0 (i.e., $\frac{\partial g(Y)}{\partial Y} > 0.$) An example of a homogeneous and a homothetic production function would be as follows



As example of a homothetic function would be:



Notice that the homothetic function has the same shaped isoquants (homotheticity says nothing about substitution effects along an isoquant, (and hence, homotheticity says nothing about the shape of isoquants); homotheticity says only something about the labeling of the output associated with each K,L set of inputs.

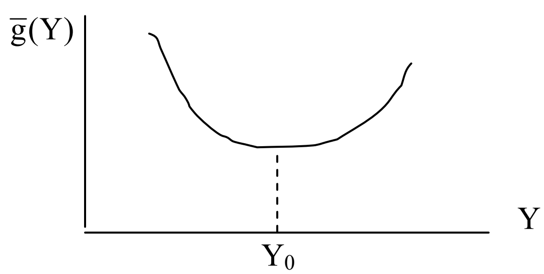
Since the first order cost minimization conditions are $g(Y) = B_L L = B_K K$, the cost function will be

$$C = g(Y) * (w/B_L + s/B_K)$$

In the short run, the competitive firm will be selling somewhere along its marginal cost curve (namely, where C'=P). In the long run, entry of new firms will continue until no firms are making profits (only a market return to labor and capital) so no more firms have an incentive to enter the market. That is, in the long run, the firm will be operating where the C' curve intersects the $\overline{C}$ curve at the point of minimum average costs.

$$\overline{C} = \frac{C}{Y} = \frac{g(Y)}{Y}\left(\frac{w}{b_L} + \frac{s}{b_K}\right) = \overline{g}(Y)\left(\frac{w}{b_L} + \frac{s}{b_K}\right) \text{ where } \overline{g}(Y) = \frac{g(Y)}{Y}$$

Suppose that Y_0 is the level of output at the firm level that minimizes $\overline{g}(Y)$ as illustrated below:



In long-run equilibrium, price will be equal to the marginal cost will be equal to the minimum point on the average cost curve, so

$$P = C' = \overline{C} = \overline{g}(Y_0)\left(\frac{w}{b_L} + \frac{s}{b_K}\right)$$

Since $L=g(Y)/b_K$ for the firm, the aggregate demand for labor with n firms will be

$$L^a = \frac{n\, g(Y_0)}{b_K} = \frac{Y^a\, g(Y_0)}{Y_0\, b_L} = \frac{g(Y_0)\, Y^a}{Y_0\, b_L} = \overline{g}(Y_0)\frac{h(D, \overline{g}(Y_0)(\frac{w}{b_L} + \frac{s}{b_K}))}{b_L}$$

Given the assumption in the graph immediately above, $\overline{g}(Y_0)$ is a number. So aggregate labor demand here is--except for the scalar $\overline{g}(Y_0)$ --the same as in the CRS case first worked through above. Indeed, nothing has changed in relative terms—any percentage change in this homothetic case (around the long-run equilibrium value Y_0) would be the same percentage change as in the CRS case above. The reason for this is that long run equilibrium occurs at points that are locally CRS (the smallest value on the average cost curve occurs in a flat, CRS-like, region).

V. **Long-run demand for labor: no substitution and non-Homothetic production:**

The MRS between capital and labor is no longer constant for any given capital-labor ratio. An example, still with no substitution between inputs would be

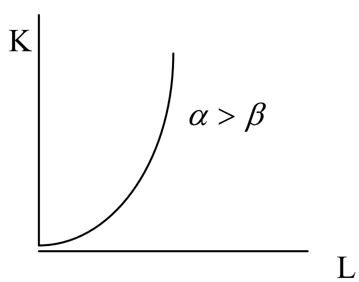
$Y = \min(L^{\alpha}, K^{\beta})$ where $\alpha, \beta > 0$ *and* $(1-\alpha)(1-\beta) < 0$ so that alpha (or beta) is larger than one, while the other parameter beta (or alpha) is less than one. Again, costs are minimized when $Y = L^{\alpha} = K^{\beta}$.

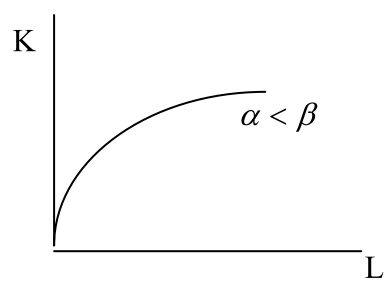
The cost function can no longer be written as a separable function of output and input prices. But there are some other, new characteristics of the isoquants that are worth talking about.

The *expansion path* is the set of cost minimizing input combinations as Y increases, holding relative prices constant. Homothetic models have linear expansion paths (how do we know this?), which is why the cost functions can be separated into a scale part and an input price part. Non-homothetic functions have non-linear expansion paths. In our example here, the first order conditions imply that

$K = L^{\frac{\alpha}{\beta}}$ (which is the equation of isoquant corners for this model)

So K=0 iff L=0. Unless $\alpha = \beta$, the expansion path is not a straight line, but differentiation shows the following will hold:





Now, even though the isoquants are still right angle so that there is no factor substitution, the K/L ratio is not really fixed. As factor prices change, this will cause the firm to move along the expansion path (the cost minimizing locus) and K/L changes. There appears to be a sort of factor substitution even with right angle isoquants.

Now let's go from this apparent factor substitution to the continuous isoquant case where there is factor substitution even when output is held constant.