

## I. HUMAN CAPITAL ACCUMULATION AND THE LIFE CYCLE DIAGRAM

Human capital, both general and firm specific, continues to accumulate after one leaves school, and this accumulation changes the employee's real wage. In this lecture, we start by considering some implications of these changes.

Human capital accumulation generally follows an inverted U shaped pattern. Investments in human capital are costly, both in terms of foregone wages as well as out of pocket expenses. Hence, workers will be willing to invest more in their human capital when the rate of return is higher or the longer the period during which returns from the investments can be realized. Human capital theory explains why workers seldom quit their jobs to return to college later in life (to give up their jobs to do so would make it very expensive). Going to college when younger is better both because the potential wage rate is relatively low so that the wages foregone due to college attendance are relatively modest and because the whole working life will be relatively longer so that the costs of the investment can more readily be recovered.

For the same reasons, the largest on-the-job training investments will tend to take place in the first years of employment. As workers age, years of employment increase and wages increase. Returns to investments then fall, because it is more expensive to take time off from work to invest in skills and the remainder of the working life is small. In the years just before retirement, the impact of depreciation of skills (customers die, and accounting systems change) may outweigh new investments, in which case the stock of human capital may decline.

Thus, human capital increases early in ones working career, probably peaks sometime late in the prime age working years, and possibly declines near retirement. Since the wage rate represents the monetary return to human capital, we expect the wage rate to follow the inverted U-shape path. To capture these life cycle changes in human capital and wages, consider a stylized working career consisting of three periods: youth, prime age, and older age periods. We assume that the worker has maximized his utility over his lifetime, having made from the beginning his decisions about how much to work each period given his choice of human capital investments and wages. All current wage income and other period wage income is assumed to be represented in discounted, present value dollar terms. We assume that wages in the youth period are \$8,000; wages during prime age are supposed to be about \$30,000, and wages during mature years are \$20,000.

The wage rate is smallest in the first period and largest in the prime age period, and the worker knows this in advance. Since these are anticipated wage changes, and hours of work decisions are made simultaneously over the life cycle, lifetime income is known from the beginning. So, anticipated increases in wages as human capital increases, generate no additional income effects as their are realized. There is only the intertemporal substitution effect that leads the worker to work more when her wage is highest, and work less when the wage is lower. Hence, hours of work are greatest when the wage rate is highest in the prime age period. As a result, discounted present value of wage income is smallest in the youth period and greatest in the prime age period.

The U shaped wage path generated by human capital accumulation, therefore, also leads to a U-shaped total wage earnings path. If these annual wage responses are plotted against the worker's age (where they are now several ages, not just the generic three discussed above), then we get Figure 1. This is an especially useful way to look at human capital generated wage changes when considering the impact of social security and private pension benefits on savings and retirement.

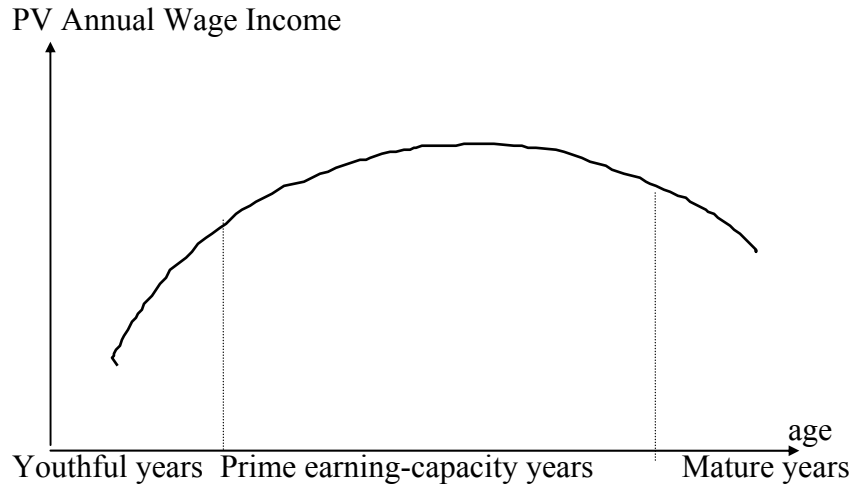


Figure 1

### Diminishing Marginal Utility of Income

When we consider how a change in the social security program or an employer's private pension program is going to affect the savings and retirement behavior of workers, we need to know how income is received over the working life, and then how that income is spent since savings is just the difference between income received and expenditures. The model of wages over the life cycle given in the last section tells us how income is received over time: typically, there will be an inverted U-shaped pattern of wage income. But since savings is just the difference between income and consumption, a theory of savings requires a theory on how money will be spent over the life cycle.

Life cycle consumption theory is a simple extension of the *convexity* principle that we discussed earlier (with respect to the utility function of workers); here the concept of convexity is applied to bundles over time rather than consumption between goods and leisure at a moment in time. Recall that the convexity assumption of bowl shaped indifference curves was nothing more than the idea that workers preferred some leisure and some goods, rather than a lot of leisure and little goods, or a lot of goods and little leisure. That is, the "convexity assumption" is an assumption that workers generally prefer moderation over extremes in consumption. The principle of moderation over the life cycle suggests that workers would prefer the same level of real consumption every year, over a pattern in which they get a lot of consumption in one year at the expense of only getting a little consumption in another year. Another way to say this, is that \$10,000 in extra consumption this year does not necessarily fully compensate me for a (discounted) \$10,000 loss in consumption next year. The value of each additional

consumption dollar diminishes with income--that is, there is diminishing marginal utility of consumption.

For example, suppose that wage income from three years were respectively \$15,000, \$40,000 and \$35,000. Total wage income from all three periods is \$90,000. Suppose that the utility from different levels of consumption were diminishing, as described below:

**Table 1: Consumption and Utility**

Consumption Level	Total Utility	Marginal Utility
\$5,000	20,000 utils	-----
\$10,000	30,000 utils	10,000 utils
\$15,000	39,000 utils	9,000 utils
\$20,000	47,000 utils	8,000 utils
\$25,000	54,000 utils	7,000 utils
\$30,000	60,000 utils	6,000 utils
\$35,000	65,000 utils	5,000 utils
\$40,000	69,000 utils	4,000 utils
\$45,000	72,000 utils	3,000 utils

Notice that more consumption is always preferred to less (consistent with our initial assumptions about preferences): as consumption increases, so does the pleasure of having spent that income (measured in “utils”). However, the utility gain when going from \$30,000 to \$40,000 worth of consumption (9,000 utils) is not as great in magnitude as the utility loss in going from \$30,000 to \$20,000 (13,000 utils). This is true at all levels of consumption: the magnitude of utility gained from additional consumption is less than the magnitude of utility lost when consumption is decreased by the same amount. This is diminishing marginal utility of consumption. We assume that peoples’ preferences have this property: the value they place on an additional dollar of consumption falls as consumption increases. Dimishing marginal utility of income just says that the second derivative of the utility function (w.r.t. income ) is negative:

$$\frac{\partial U}{\partial I} > 0, \text{ marginal utility is positive; but}$$

$$\frac{\partial^2 U}{\partial I^2} < 0, \text{ but marginal utility increases at a decreasing rate.}$$

When preferences have this diminishing marginal utility property, utility can always be increased by smoothing consumption out over the life cycle. For example, if the worker consumed exactly what he earned in the current period, then the total utility would be  $39,000 + 69,000 + 65,000 = 173,000$  utils over one's life. This can be improved, however. Since an additional dollars' worth of consumption has more utility value at \$15,000 than it does at \$40,000 or at \$35,000, overall utility can be improved by transferring consumption income from the last two periods to the first period. This should continue until the transfers no longer increase utility, which will be when consumption is equalized in all three periods at \$30,000. Then total utility is 180,000 utils, a preferred outcome for the worker. This is a general result: with diminishing marginal utility of consumption, the worker (household) will want to equalize consumption over time. Consumption will be the same in all periods.

Formally (we ignore discounting in this example), the solution to the problem of choosing consumptions values,  $C_i$ , in year  $I$ , is to:

$$\max L \sum_{i=\text{years}} U(C_i) + \lambda (C_T - \sum_{i=\text{years}} C_i)$$

which yields the solution  $U'(C_i) = \lambda$ , which implies that consumption will be equal in all periods. (Why does it imply this?)

### Savings During the Life Cycle

Since we have a theory about how wage income is generated, and a theory about how that income will likely be spent, putting the two together can provide insight into savings behavior. We initially simplify the model by assuming that the retirement decision has already been predetermined and the date of death is known in advance. Then, since workers want to even out their consumption over the life cycle, there will be periods of savings and dissavings as indicated in Figure 2.

In the youth years, desired consumption will exceed income and workers will borrow against future earnings. This borrowing may take many forms—school loans, "loans" from family members, home mortgages, etc. As wages continue to increase through the prime age years, eventually wage income exceeds consumption levels. During this period—marked in Figure 5.7 as the savings period—loans are paid off from earlier years and savings towards retirement accumulates. Finally, in the retirement years, the remainder of savings from the prime age years are consumed. The shaded area above the consumption line in the prime age years during which savings takes place will equal the two shaded areas below the consumption line, periods in which savings are drawn down. If we define "**gross savings**" as the amount of wage income that is not consumed in the current period (or not used to pay off prior debt), then "gross savings" will be that part of the prime age period savings that is not used to pay off the youth period debt. In the absence of a bequest motive and uncertainty about death/retirement, it will equal the shaded area below the line in the older age period. (Why?) Also in the absence of bequests or uncertainty, net savings over the whole life cycle will be zero.

Annual Wage

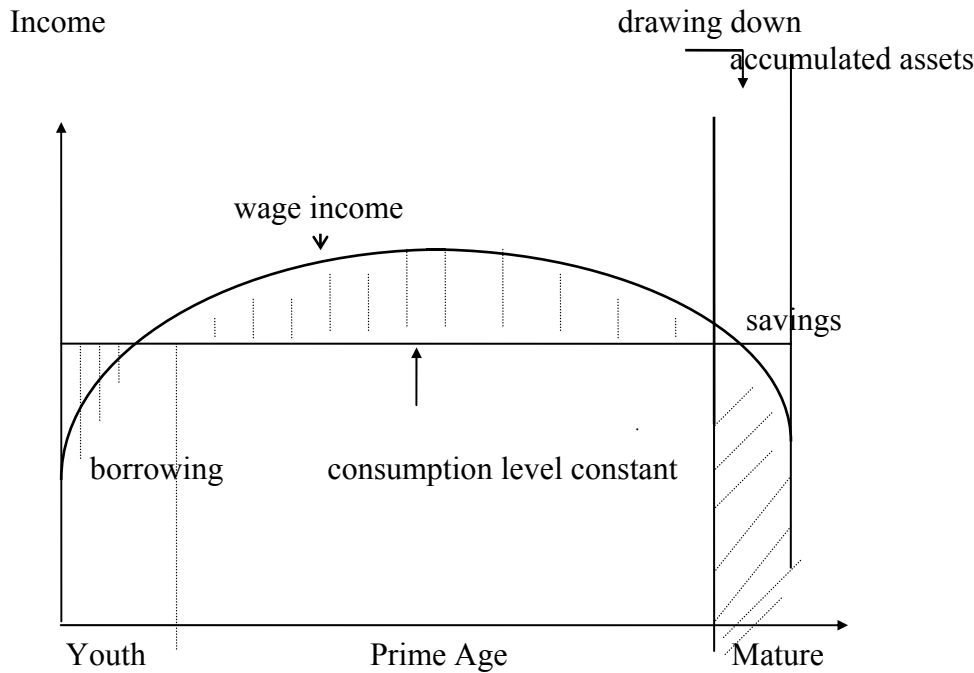


Figure 2

A worker gets interest payments for depositing their "gross savings" in a financial intermediary (say a bank). These savings are, in turn, made available for businesses who want to fund new investments. Hence, an adequate amount of savings is essential for investments and growth in the economy. It is useful, therefore, to consider what happens to gross savings as non-wage income or the wage rate changes.

Social security benefits may reduce the level of private gross savings and change retirement incentives. The argument that social security will reduce private savings in the economy hinges on social security benefits being viewed as substitutes for private savings for retirement. If this is the case, savings in the prime age years can fall by the amount of expected social security benefits. The problem is that those social security taxes that the employee contributes into the system in the current year are not really "saved" anywhere but simply transferred to current retirees. Those taxes are not "set aside for future use" in the sense that private savings are, and do not contribute to the pool of savings available for new investments in the economy. Hence, the current form of "pay as you go financing" in social security may in fact hinder economic growth.

## II. The Retirement Decision in the Life Cycle Model

Is it best to retire at age 62:

$$PV_{62} = B_{62} + \frac{B_{63}}{(1+r)} + \frac{B_{64}}{(1+r)^2} + \frac{B_{65}}{(1+r)^3} + \frac{B_{66}}{(1+r)^4} + \frac{B_{67}}{(1+r)^5} + \frac{B_{68}}{(1+r)^6} + \frac{B_{69}}{(1+r)^7} + \text{etc}$$

and get Social Security and pension benefits, B, each year; or is it better to continue to work and retire at age 70:

$$PV_{62} = W_{62} + \frac{W_{63}}{(1+r)} + \frac{W_{64}}{(1+r)^2} + \frac{W_{65}}{(1+r)^3} + \frac{W_{66}}{(1+r)^4} + \frac{W_{67}}{(1+r)^5} + \frac{W_{68}}{(1+r)^6} + \frac{W_{69}}{(1+r)^7} + \text{etc}$$

and get wages each year until retirement, and get increased annual retirement benefits thereafter.

How social security and private pension benefits work.