Econ 463 R. Butler 2005 Lecture 4

I. Introduction

Models of family labor supply usually involve defining something known as the family utility function, and just applying the standard budget constraints on it. The theoretical weakness of this (the most common) approach is explaining how the family utility function was determined in the first place. How did we get from individual maximization to a family utility function? The answer is: its hard to do except under very strong assumptions.

So we either just assume that a family utility function exists (and maximize it), or we have the family members playing games with each other that results in utility functions of peculiar types. Neither approach seems to dominate economists' luncheons yet. But every economist does agree that, whatever is the best description of the process, that family production and family decision making is very important. (So while this last sentence is not exactly bringing the scriptures into the classroom, its darned close. Remember that.)

We, too, begin by assuming that households do have a utility function that they maximize. We examine how household time (and the time-intensity of goods) enters into the decision making process in section II. We call this the commodity-space view, because we act as if the household's utility is defined over the commodities. In section III, we take an input-space view; utility is now defined over the inputs used. In the next lecture, we look at the life cycle, with particular attention to retirement and savings. Please note in this lecture we are letting

T_1,T_2=time spent in producing household commodities in section II
H=time spent in household production in Section III ($=T_1$)
N=time spent employed in market work (that is, it plays the same role that the "H" did of previous sections)

II. Time allocation with two commodities(Z_1,Z_2)

1) $U = U(Z_1,Z_2)$ utility function

2) $Z_1=f_1(X_1,T_1)$ home production function (CRS) for Z_1

3) $Z_2=f_2(X_2,T_2)$ home production function (CRS) for Z_2

where Z_i=commodity i
X_i=goods used to produce commodity i (with price P_i)
T_i=time input in producing commodity i
N=time spent in market work (hours work)

The time constraint is

$$T = T_1 + T_2 + N$$

and the money constraint is

$$V + WN = P_1X_1 + P_2X_2$$

These two constraints can be reworked to get the full income constraint:

$$4)\ V + WT = P_1X_1 + WT_1 + P_2X_2 + WT_2$$

So total income (V + WT) is spent in the production of the Z_i commodities. The total cost of producing Z_i is given as

$$5)\ C_i = P_iX_i + WT_i$$

which implies an average cost of

$$6)\ C_i/Z_i = P_{i(}X_i/Z_i) + W(T_i/Z_i)$$

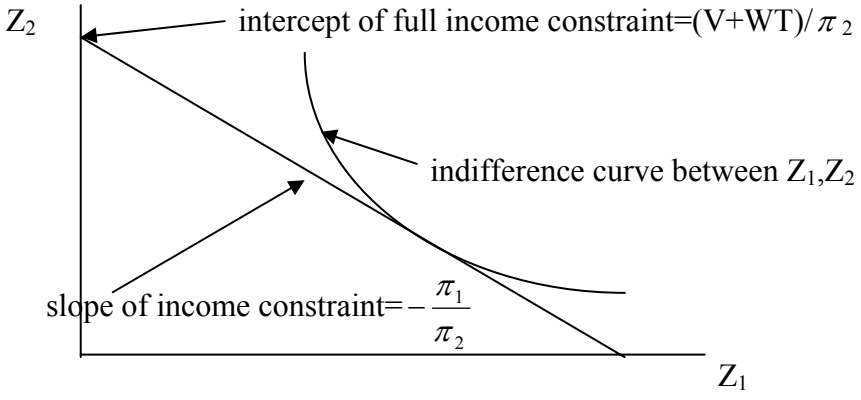
If the production functions for Z_1 and Z_2 exhibit constant returns to scale (as we assume), then the average cost of producing Z_i is equal to marginal cost (or shadow price), denoted as π_i so that

$$7)\ V + WT = [P_1(X_1/Z_1)+W(T_1/Z_1)]Z_1 + [P_2(X_2/Z_2)+W(T_2/Z_2)]Z_2$$

$$\text{or } V + WT = \pi_1Z_1 + \pi_2Z_2$$

So we maximize (1) subject to (7), and get sort of the usual analysis that we all know and love.

Apply the usual static analysis to our home production model:



$$\frac{W \cdot T_1}{\pi_1 \cdot Z_1} > \frac{W \cdot T_2}{\pi_2 \cdot Z_2}$$

Using what we know about percentage changes from before, we find that

$$9)\ \text{dln } \pi_i = \frac{P_i \cdot X_i}{\pi_i \cdot Z_i} \text{ dln } P_1 + \frac{W \cdot T_i}{\pi_i \cdot Z_i} \text{ dln W}$$

Can this family production model explain the fall in family size?

III. Specializing to Generic Home Production, as seen from Input Space

In terms of a household production model, what does indifference curves and constraints look like in (T,X)-space (i.e., from the point of view of the inputs) rather than in (Z_1,Z_2)-space (from the point of view of the commodities)?

Indifference curves are derived from utility maximization, where here we'll assume that there is only one Z that individuals desire (in our community, that would be child rearing) so that U=U(Z(T,X)). Now take the total differential

$$10)\ dU = \frac{\partial U}{\partial Z}\cdot\frac{\partial Z}{\partial X}dX + \frac{\partial U}{\partial Z}\cdot\frac{\partial Z}{\partial L}dL$$

which (recall that along an isoquant dU=0) we can solve to find the slope of the indifference curve

$$11)\ \frac{dX}{dL} = -\left(\frac{\frac{\partial Z}{\partial L}}{\frac{\partial Z}{\partial X}}\right)$$

which gives us the usual convex indifference curves for the usual reasons (relating to diminishing marginal productivity).

Now suppose that in addition to purchasing X in the market place (suppose that X are disposable diapers), we can also produce X by using our own time (we can rise cotton in our garden, than weave some real diapers, then recycle them through our own laundry). This is subject to diminishing returns:

12) $X_h=f(H)$

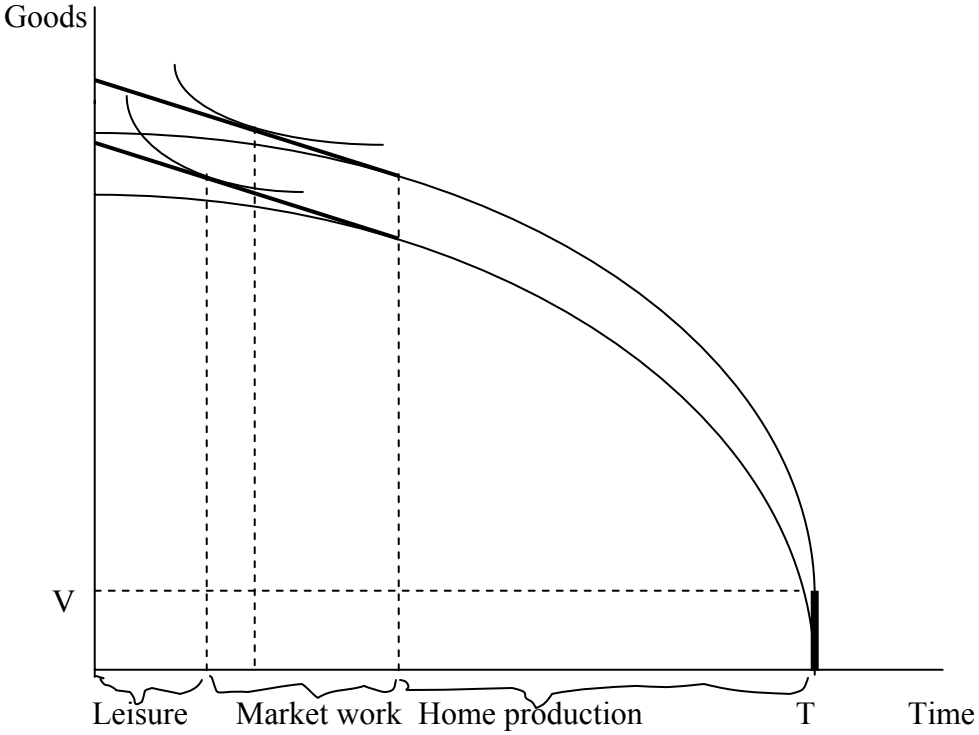
where the production function is such that f'>0 and f''<0, i.e., its concave. The variable definitions and constraints are now

$X = X_h + X_m$, where X_h=X produced at home and
X_m=X purchased in the market

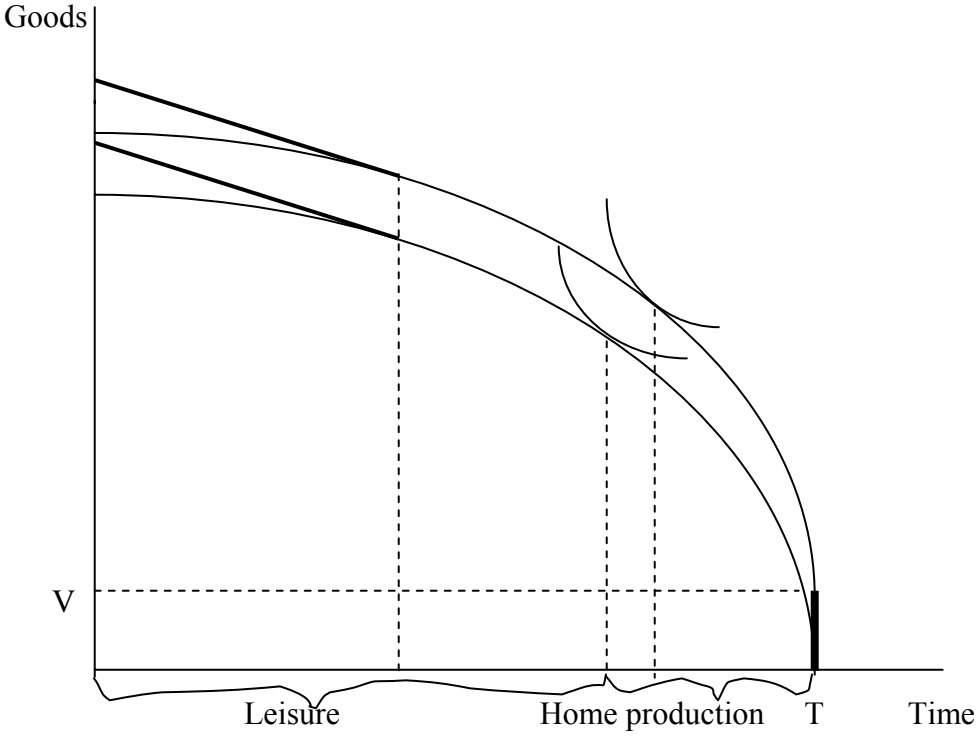
$T = L + H + N$, where N=time at market work
L=time in producing Z
H=time spent in producing X_h

The picture for all of this is given in graph below, where the concave part of the constraint moving left to right from T is due to the concavity assume in equation (12) above, and the linear proportion starting at A_i (i=0,1) is just where we go to the market to purchase our inputs. In these first two graphs we show a pure change in income:

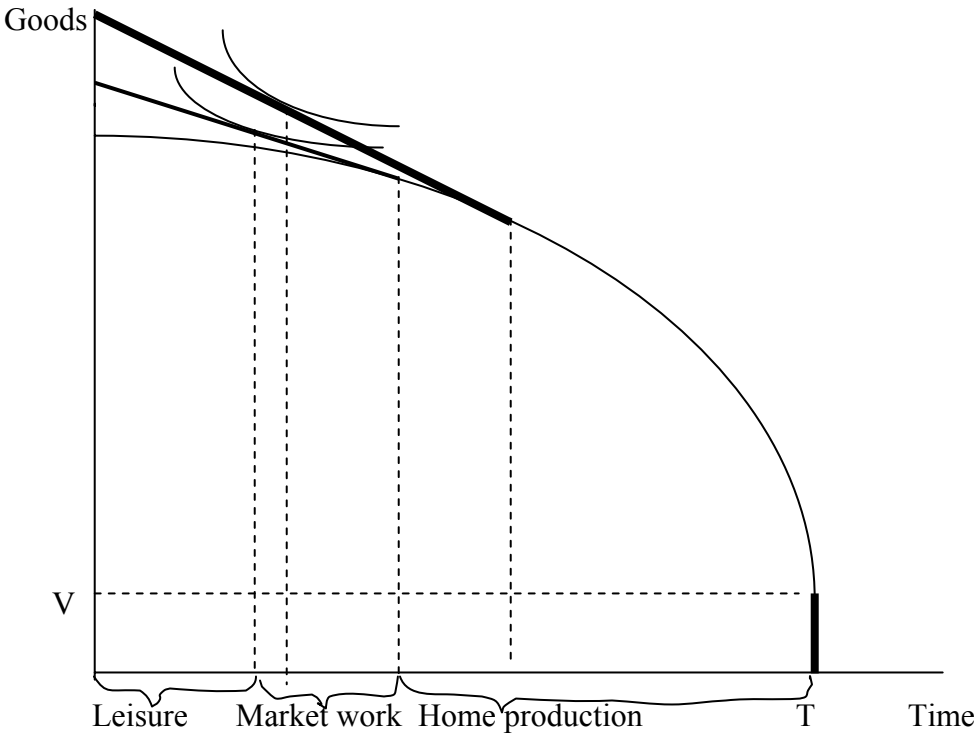
The Some-Market-Work Graph (Relatively Large Demand for Goods)



The No-Market Work Graph (Leisure is a Pretty Good Thing)



The implications of a rise in the wage rate:


Goods
V
Leisure
Market work
Home production
T
Time