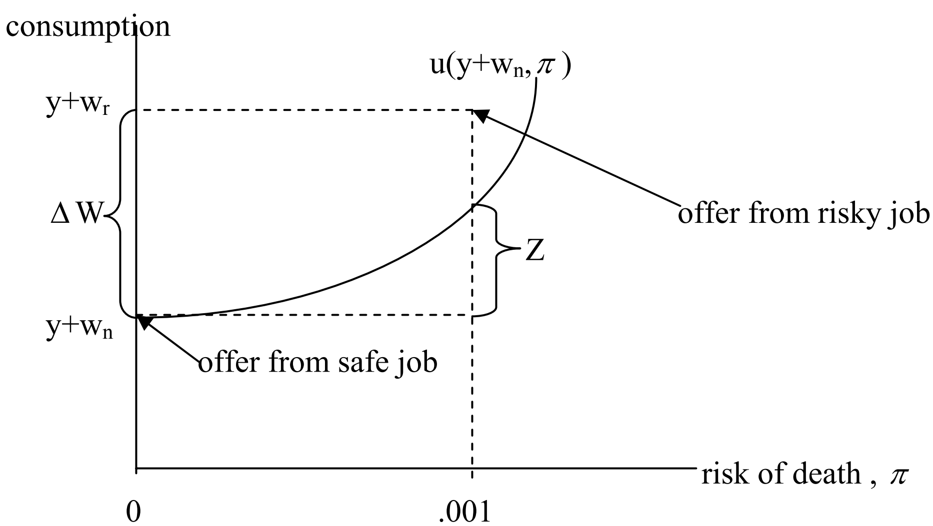
Econ 463 R. Butler 2005 Lecture 12
(((this lecture borrows a lot from Rosen's "Theory of Equalizing Differences" in the Handbook of Labor Economics)))

I. Risk: Some Simple Introductory Models

I. Going Postal: What would you choose:

Federal Express offer ($12/hour, fringes, no risk of death)
United States Postal Service offer ($12/hour, fringes, risk of death=1 in a thousand/yr)

2. 10,000 Cloned Yous and Market Equilibrium: One You and Two Types of Jobs: Suppose that you were cloned. You (the risk averse worker) would want a compensating wage for risk of death; wages would adjust. What would determine the equilibrium difference in wages? relative entry into the two businesses...the wages in FEDX may drop those in USPS would rise until you (the marginal worker) were indifferent between the two.



Two hypothetical wage and risk offers; the worker compares the compensating wage in the market (ΔW) with the compensating variation (Z=the additional compensation necessary to make the worker indifferent between the two offers, given the utility index generated by the safe job).

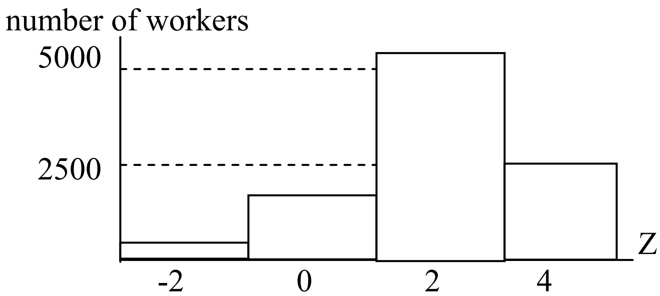
In this market, what would the risk averse you do?
What would happen to the compensating wage if everyone were just like you?

If the market compensating difference is $2/hour; that amounts to $2x2000=$4,000 a year. Among 1000 workers, one will usually get killed. So collectively, the value of that life is 1000x$4,000=$4,000,000.

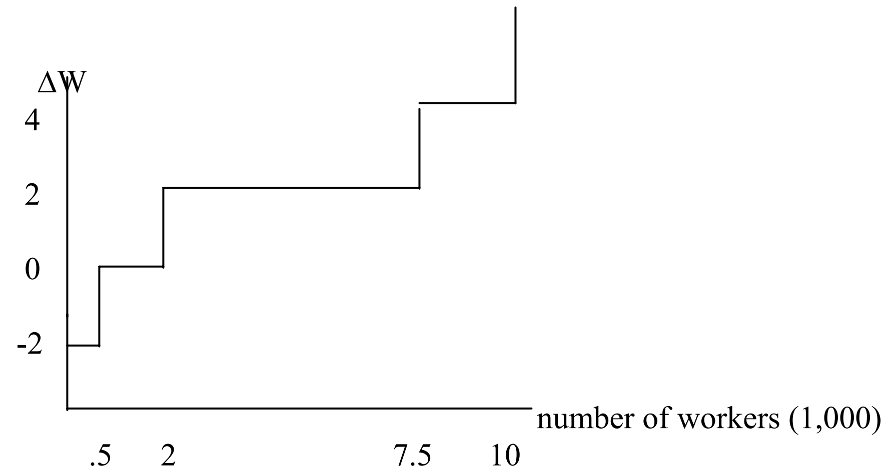
3**. The Duplicitous You and Market equilibrium: One Type of Job, but Varying Tastes for Risks among workers**. When there is no variation in the worker's supply of risky employement, but there is variation in the demand for risky employment from firms (the example above), market equilibrium was achieved by the establishing market pay differentials (price equilibrates because of a perfectly elastic supply of labor). Now we have only one demander of risky employment (so the demand curve for risky employment is inelastic supply), but differences in the suppliers of risky employment. In this alternative world with only one risky job (at the postal service, the alternative is to engage in non-risky, non-market production), there are many different types of potential risky employment that could be supplied by the workers:

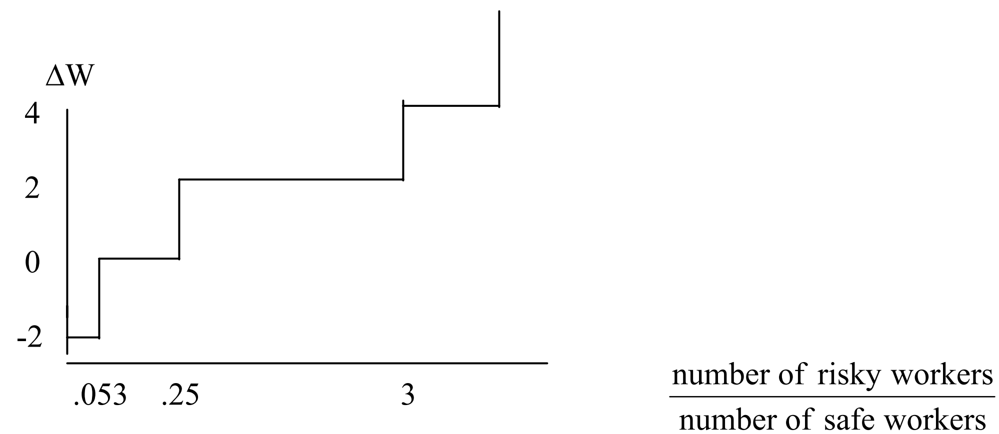
500--willing to work at risky USPS for Z= - 2 (risk lovers) even with $12 offer at FEDX
1500--willing to work at risky USPS for Z=0 (almost risk neutral depending on theology)
5500--willing to work at risky USPS for Z=2 (risk averse)
2500--willing to work at risky USPS for Z=4 (way risk averse)

The distribution of tastes for risks looks like:



If the compensating wage were $2/hour, then up to 7,500 workers would be interested in the job (at least indifferent); when Δ W = $2, the relative supply of risky employment is 7500 jobs. When Δ W = $0, the relative supply of risky work is 2000 jobs, etc. We can plot the relative supply of risky employment using either the absolute number of workers (or the relative number=ratio of risky job takers to non-risky job takers given ΔW), as follows:

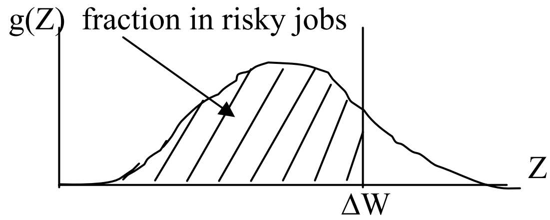




IN this example, the supply of risky jobs=5,000 (the postal service is offering 5000 jobs, so that that relative supply is 1 since there are 5000 risky jobs and 5000 in the safe home-production sector). From either the absolute demand curve (the upper graph immediately above) or the relative demand curve (the graph immediately above), the compensating wage difference will be 2. That is, the exogeneously determined supply of jobs (in this case, by the postal service) determines the relative wage as we shift along the demand curves above.

II. Variations in Tastes for Risk (to Workers) and Benefits from Risk (to the employers); but the degree of RISK IS FIXED (that is, the only two outcomes are 1) no risk of death, or 2) probability of death=.001)

Now consider a somewhat more realistic setting in which we have **variation in both the supplies and demanders of risky employment,** where equilibrium is achieved by changes in the compensating wage and relative numbers in each occupation. Let subscript1=those in the risky occupation (where the probability of death=.001) and subscript 0=those in the "safe" occupation (with no fatal injury risk). Suppose that the distribution of compensating variations for risk **across workers** looks like



with density function, g(Z) and distribution function G(Z). G(Z) and g(Z) are related in the usual way, where

$$\int_0^{\Delta W} g(z)dz = G(\Delta W)$$

the integral means that we add the fraction (g(z)) of workers with tastes for risk over all values of Z less than Δ W. This is the number of workers willing to work in the risky jobs, and this cumulative fraction (less than one) is denoted as G(Δ W). Hence, the relative supply of risky employment is given as:

$$N_1/N_0 = G(\Delta W)/[1-G(\Delta W)]$$

For Firms: The benefit from allowing risk in the workplace is the additional output that results. Risk of death is a byproduct of production here, and lowering all workplace risk comes at a cost given that we have already taken advantage of any inefficiencies that we could have gotten rid of in the process of organizing production. That is, our profit maximizing firm will not be operating in a region where one can increase safety (reduce workplace risk) without cutting back any output. (Why?) This additional value added is just

1) $MP_{risky} P - MP_{safe} P$

We assume that the product price (P) is fixed at one. We assume that the production technology is Leontiff, so with first order condtions:

$Y=a_1L$ for work in the risky sector, and $Y=a_0L$ for work in the safe sector.

The marginal products are constant (=a_1,a_0) given the production functions), the benefit from allowing job risk is just

2) $a_1 - a_0 = B$

So if the ***benefit from allowing risk*** is greater than the compensating wage paid to workers, then you allow job risk. That is allow risk if $B > \Delta W$.

Otherwise if $B < \Delta W$, you invest in job safety, because it is cheaper to do so. If we let f(B) be the distribution of B (across the firms) , then the fraction of firms offering a safe job will be

$$\int_0^{\Delta W} f(B)dB = F(\Delta W) \quad \text{safe job types}$$

while the fraction of risky job offers will be $\int_{\Delta w}^{\infty} f(B)dB = 1 - F(\Delta W)$.

So $N_1/N_0 = [1 - F(\Delta W)]/ F(\Delta W)$

Supply, demand and market equilibrium together.

<u>**A Specific Example**</u> <u>SUPPLY:</u> I begin by assuming that as we look across workers, with their varying tastes for risk in the workplace (z), that we can adequately characterize their tastes with an exponential distribution:

3) $G(\Delta W) = 1 - \exp(-\Delta W/\beta)$ the fraction of workers choosing risky jobs

which is the fraction of workers choosing risky jobs since the compensating differential ΔW is larger than that required (namely z) to induce them to work.

(The exponential distribution is a one parameter distribution with mean equal to β.

That is, $\beta = \int_0^{\infty} z\, g(z)\, dz$, the expected value of the exponential random variable. This is the average value of tastes for risk.

Hence the fraction choosing safe jobs is just

4) $1 - G(\Delta W) = \exp(-\Delta W/\beta)$

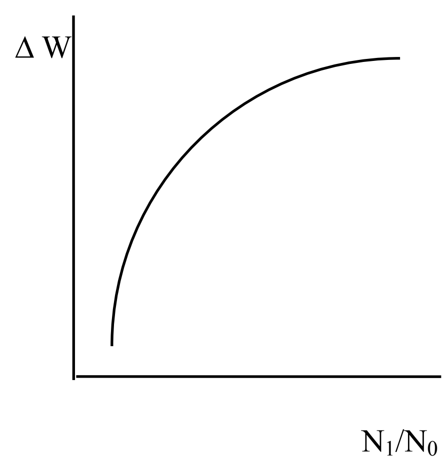
We want to express relative employment (N_1/N_0) as a function of the wage differential ΔW, so we form the ratio

5) $\dfrac{N_1}{N_0} = \dfrac{1 - \exp(-\Delta W/\beta)}{\exp(-\Delta W/\beta)} = \exp(\Delta W/\beta) - 1$

which we can also express as

6) $\Delta W = \beta \cdot ln\left(\dfrac{N_1}{N_0} + 1\right)$

By taking the first and second derivatives (of ΔW with respect to N_1/N_0-- what are they?), we see that the supply function has the shape that is indicated in the figure on the right.



<u>DEMAND:</u>

To keep the analysis simple, we also assume that the Benefit from allowing job risk (B) is distributed as an exponential, so that

7) $F(\Delta W) = 1 - \exp(-\Delta W/\gamma)$ the fraction of safe firms

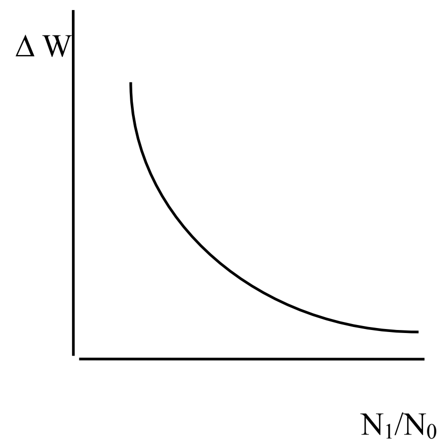
is the number of firms for whom B ($=a_1-a_0$) is less than the compensating wage ΔW so that it is cheaper (in terms of lost output) to have a safe production environment than it is to pay the extra wages associated with a risky workplace. That is, you allow for a risky workplace iff $a_1-a_0 > \Delta W$, and a clean workplace iff $a_1-a_0 < \Delta W$. Using the exponential distribution, we can also solve for a relative demand curve, expressed as follows:

$$8)\quad \frac{N_1}{N_0} = \frac{\exp(-\Delta W/\gamma)}{1-\exp(-\Delta W/\gamma)} = (\exp(\Delta W/\gamma) - 1)^{-1}$$

or in an equivalent form

$$9)\quad \Delta W = \gamma \cdot ln\left(\left(\frac{N_1}{N_0}\right)^{-1} + 1\right)$$

By taking the first and second derivatives (of ΔW with respect to N_1/N_0--do it), we see that the demand function has the shape that is indicated in the figure on the right.



<u>EQUILIBRIUM</u>:

By equating supply and demand, we can find (ΔW, N_1/N_0)-pairs that satisfy both curves simultaneously and thus represent market equilibrium points. For example, by equating ΔW_s with ΔW_d from equations 6 and 9 respectively, we can solve for N_0 and N_1 as follows:

$$10)\quad \left(\frac{N_0}{N_0+N_1}\right)^{\gamma} = \left(\frac{N_1}{N_0+N_1}\right)^{\beta}$$

From this its clear that if $\beta = \gamma$ then $N_0=N_1$, and if $\beta > \gamma$ (which means that the workers have relatively higher tastes for safety, so fewer workers take the risky jobs) then $N_0>N_1$ (more work in safe firms). And if $\beta < \gamma$ so that there are low tastes for safety (i.e., my boy scout troop), then $N_0<N_1$ (more work in risky firms).

We can also solve 5 and 8 for Δ W; Δ W is equal to ln(2) if $\beta = \gamma$, but is difficult to solve for explicitly otherwise (you could simulate solutions with a computer). Δ W are implicitly solved as a function of β, γ from the following equation:

$$11)\ \exp(\Delta W(\frac{1}{\beta} + \frac{1}{\gamma})) = \exp(\Delta W/\beta) + \exp(\Delta W/\gamma)$$

Note that β and γ are the means of their respective distributions (so that a higher β is associated with higher tastes for safe working conditions, and a higher γ indicates greater gains to the firm from being safe). What happens to the supply and demand curves as these parameters (β, γ) increase? What happens to the equilibrium Δ W and the mix of workers between safe and risky jobs?

We can extend the model to discuss the economic rents gained by the intra-marginal workers. The rent for those in the risky jobs is just the difference between the compensating wage difference and Z, or Δ W - Z. Hence, the average rent for those choosing a risky job is Δ W - E(Z|D=1), where "E(Z|D=1)" is the expected value of Z for those choosing the risky jobs. The expression E(Z|D=0) is the expected value of Z for those in the safe jobs, so that the rent there is E(Z|D=0)- Δ W. "E(Z)" would be the overall mean of the distribution, for both those in risky and those in safe jobs.

Given our exponential example, what are the average rents received by those choosing risky jobs, and by those in safe jobs? Hint:

$$E(Z|D=1) = \int_0^{\Delta W} z g(z) dz = \int_0^{\Delta W} z \cdot \frac{\exp\left(-\left(\frac{z}{\beta}\right)\right)}{\beta} dz = \beta[1 - (\exp(\frac{-\Delta W}{\beta})(\frac{\Delta W}{\beta} + 1))]$$

just for your reading pleasure. A similar expression can be made for E(Z|D=0), here we used the indefinite integral $\int x \exp(ax) dx = \frac{\exp(ax)}{a^2}(ax - 1)$.