



## II. Some Concepts

11. Suppose that two random variables are constructed from a flipping a fair coin twice. Define  $Z$  to be a random variable whose value equals the number of heads in two flips (so  $X = 0, 1$  or  $2$ ). Define  $W$  to be the random variable whose value equals one if the two flips get the same results ( $Y=1$  if the experimental outcome is either 'heads, heads,' or 'tails, tails'; and  $Y=0$  otherwise).

A. Fill in the joint probability density function for the following table (i.e., indicate what the joint probabilities of each of the outcomes are):

	$Z=0$	$Z=1$	$Z=2$
$W=0$			
$W=1$			

B. calculate the marginal probability densities  $f(Z)$  and  $f(W)$

C. Calculate  $E(Z)$  and  $V(Z)$  (no credit unless you show the right formulas).

D. Calculate the conditional probability density  $f(W|Z=1)$  (again, no credit unless you show the right formulas)

E. Are  $W$  and  $Z$  independent? Why or why not?

12. Prove that under the usual model assumptions that the least squares estimator,  $\hat{\beta}$ , is unbiased and has a covariance matrix equal to  $\sigma^2 (X'X)^{-1}$ .

13. Assume that “schooling size” has no negative impact on student performance in standardized math tests, where the null hypothesis is that enrollment (ln enroll) has no effect on math10 scores and the alternative hypothesis is that it is not good for math10 scores..

- a. What is the mathematical way of stating the null hypothesis and the alternative hypothesis?
- b. What is the probability of making a type-II error assuming i) that the critical value of the type I error is 5 percent, and ii) that the true coefficient on the ln(enrollment) variable is
  - a. -1.5?
  - b. -1.8 ?

given the following:

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO	PARTIAL P-VALUE	STANDARDIZED CORR. COEFFICIENT	ELASTICITY AT MEANS
LTOTCOMP	21.155	4.056	5.216	0.000	0.251	9.2493
LSTAFF	3.9800	4.190	0.9500	0.343	0.047	0.7600
LENROLL	-1.2680	0.6932	-1.829	0.068	-0.091	-0.3950
CONSTANT	-207.66	48.70	-4.264	0.000	-0.208	-8.6143

14. Indicate whether the following statements are True, False or Uncertain (indicate which, you are graded only on your explanation for your answer):

a. "There is no impact from including irrelevant regressions in an empirical model (i.e., from over-specifying a model)."

b. "If all my sample observations have exactly the same level of ability, then omitting ability from the empirical specification where I regress wages on education will lead to no omitted variable bias."

- II. 15. a. Prove that the least squares estimator for the variance,  $s^2$ , is unbiased using matrix algebra.  
b. Which OLS assumptions did you use in your proof?