

YOUR NAME: _____

Section I (30 points) Questions 1-10 (3 points each)

Section II (20 points) Questions 11-12 (10 points each)

Section III (50 points) Questions 13-14 (15 points each)

Question 15 (20 points)

Section I. Define or explain the following terms (3 points each)

1. LaGrange-Multiplier test-

2. log-likelihood ratio test-

3. law of large numbers-

4. central limit theorem-

5. maximum likelihood estimation--

6. method of moment estimators-

7. dummy variable trap --

8. variance inflation factor--

9. prediction interval for forecast-

10. instrumental variables-

II. Some Fun Stuff:

11. True, False or Uncertain question: you are graded on your explanation and not on whether you guessed T or F correctly. “When regressed on a bunch of demographic factors (say age, education, and gender), there is no way to tell whether the natural log of wages or wages themselves (the linear or untransformed data) provide a better fit for the regression, since outcome (wages) are measured along different scales (log vs. linear).”

12. Suppose that in the logit model, the probability of being married, P_i , is a function of only age and a constant as follows:

$$P_i = \frac{\exp(\beta_0 + \beta_1 \text{Age}_i)}{1 + \exp(\beta_0 + \beta_1 \text{Age}_i)}$$

where people can get very old (say billions of years old) and we can count the preexistence as negative years of age (since age is time since your birth on earth). Show that even if people get old in this large negative age sense, or large positive age sense, that P_i is bounded between 0 and 1, and so avoids the boundedness problem of the linear probability model.

III. Really Fun Stuff

13. Suppose that for the standard regression model, $y = X\beta + \mu$, we rescale both the independent variables and the dependent variable by non-zero variables $c_0, c_1, c_2, \dots, c_k$ by regressing $c_0 y_i$ on a constant $c_1, c_2 x_{2i}, c_3 x_{3i}, \dots, c_k x_{ki}$ (so there are k regressors, including an intercept which is measured as c_1 instead of 1, so that the intercept coefficient is β_1). In other words, instead of the OLS estimator $\hat{\beta} = (X'X)^{-1} X'Y$, you do OLS on the transformed data $\hat{\beta}^* = (X'^* X^*)^{-1} X'^* Y^*$, where the transformed data can be described by:

$$X^* = XC \text{ where } C = \begin{bmatrix} c_1 & 0 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 & 0 \\ 0 & 0 & c_3 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & c_k \end{bmatrix} \text{ and } Y^* = C_0 Y, C_0 = \begin{bmatrix} c_0 & 0 & 0 & 0 & 0 \\ 0 & c_0 & 0 & 0 & 0 \\ 0 & 0 & c_0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & c_0 \end{bmatrix} = c_0 I.$$

Prove that the i^{th} beta between the transformed data, and the untransformed data, have the following relationship: $\hat{\beta}_i^* = \frac{c_0}{c_i} \hat{\beta}_i$.

14. Data from GPA2.RAW generated the regression below, where
 colgpa=college gpa
 hsize=size of the individual's high school graduating class (in hundreds)
 hsizesq=square of hsize
 female=1 if female, 0 if male

```
|_genr hsiz_fm=hsize*female
|_genr hsiz2_fm=hsizesq*female
|_ols colgpa hsize hsizesq female hsiz_fm hsiz2_fm
```

```
R-SQUARE = 0.0172      R-SQUARE ADJUSTED = 0.0161
VARIANCE OF THE ESTIMATE-SIGMA**2 = 0.42683
STANDARD ERROR OF THE ESTIMATE-SIGMA = 0.65332
SUM OF SQUARED ERRORS-SSE= 1763.2
MEAN OF DEPENDENT VARIABLE = 2.6527
LOG OF THE LIKELIHOOD FUNCTION = -4106.10
```

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 4131 DF	PARTIAL P-VALUE	STANDARDIZED CORR. COEFFICIENT	ELASTICITY AT MEANS
HSIZE	0.91537E-01	0.2517E-01	3.637	0.000	0.056	0.2413
HSIZESQ	-0.14978E-01	0.3453E-02	-4.337	0.000	-0.067	-0.0613
FEMALE	0.21530	0.5867E-01	3.670	0.000	0.057	0.1626
HSIZ_FM	-0.63405E-01	0.3778E-01	-1.678	0.093	-0.026	-0.1745
HSIZ2_FM	0.96083E-02	0.5201E-02	1.847	0.065	0.029	0.1459
CONSTANT	2.4951	0.3931E-01	63.47	0.000	0.703	0.0000

```
|_test
|_test female=0
|_test hsiz_fm=0
|_test hsiz2_fm=0
|_end
```

```
F STATISTIC = 17.297779 WITH 3 AND 4131 D.F. P-VALUE= 0.00000
```

a). Should hsizesq be in the regression? Why or why not?

b) Is there an optimal high school size for males? If so, what is it? If not, why not?

c) Is there an optimal high school size for females? If so, what is it? If not, why not?

d) What does the F-statistic at the bottom of the page indicate about the regression results? (Be as specific as you can in answering)

15. For the general linear regression model, $y = X\beta + \mu$, assume that the modeling assumptions are all satisfied. Further, assume that $\text{plim}\left(\frac{X'X}{n}\right) = \Omega$, a positive definite, symmetric matrix for any size n . Then show that the OLS estimators for the linear regression model are consistent.