

YOUR NAME: \_\_\_\_\_

Section I (30 points) Questions 1-10 (3 points each)

Section II (40 points) Questions 11-14 (10 points each)

Section III (30 points) Question 15-16 (15 points each)

Section I. Define or explain the following terms (3 points each)

1. numerator degrees of freedom in a F-test -

2. statistical significance vs. practical significance -

3. type II error -

4. conditional probability density function of y given x-

5. omitted variable bias-

6. adjusted R-square -

7. formulas for population variance and sample variance of a random variable -

8. homoskedasticity -

9. probability significance values (i.e., 'p-values')-

10. show that  $\sum_{i=1}^N (y_i - \bar{y})(x_i - \bar{x}) = \sum_{i=1}^N (y_i - \bar{y})x_i$

## II. Some Concepts

11. Indicate whether the following statement is True, False or Uncertain and explain why. You are graded only on the explanation for your answer. “An orthogonal projection is what happens when you add another regressor to your model that is uncorrelated with the original regressors.” (recall ‘regressor’ means ‘independent variable’)

12. Indicate whether the following statement is True, False or Uncertain and explain why. You are graded only on the explanation for your answer. “In the sample regression model,  $\hat{\beta}$  is a random variable because it is an estimate of the  $\beta$ -vector in the population model, and the  $\beta$  in the population model is random.”

13. My son has a pyramid dice, with four sides numbered from 1 to 4. Let  $W$  be the random variable corresponding to number that's on the bottom side when the dice is rolled. If the dice is not fair, but the probability that the sides with numbers 1, 2 or 3 will occur is one sixth (for each of these events taken separately) then

- a. What is the expected value of the random variable  $W$  and what is the variance of  $W$ ?
- b. If we did not know whether the dice were fair or not (i.e., that each outcome was equally probable), how could we test for that?

14. For the simple regression,  $y_i = \beta_0 + \beta_1 x_i + \mu_i$ , we can regress the dependent variable on the independent variable and get  $\hat{\beta}_1$ , as we did in problem 13. But we can also divide through the dependent variable by its standard deviation (call it  $\sigma_y$ ) to get " $y_i / \sigma_y = y_i^*$ ," and similarly divide through the independent variables by its standard deviation to get " $x_i / \sigma_x = x_i^*$ ." Then we have a model of normalized regressors:  $y_i^* = \alpha_0 + \alpha_1 x_i^* + \varepsilon_i$ . Prove that following relationship exists between the slope coefficient of the simple model and the slope coefficient of the normalized model:  $\hat{\alpha}_1 = \hat{\beta}_1 \frac{\sigma_x}{\sigma_y}$ .

15. Prove that under the usual model assumptions that the least squares estimator of the multiple regression model,  $\hat{\beta}$ , is the minimum variance estimator among all linear, unbiased estimators (that is, prove the Gauss-Markov theorem, that the OLS estimator is BLUE).

16. Prove under that standard assumptions for the linear regression model that the estimator for variance,  $s^2$ , is unbiased (that  $E(s^2) = \sigma^2$ ).