A General Equilibrium Model of Specialization and Market Development as Engines of Economic Growth

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Abstract

This paper constructs a model of growth based on Adam Smith's notions of specialization and extent of the market. We seek to explain the following stylized facts. 1) The share of household production in total output has fallen over time as the economy has grown. 2) Services as a percent of GDP have risen at the same time. In this paper growth depends on specialization of labor according to comparative advantage in production and learning-by-doing in transactions services. It is a model of sustained, but not infinite, growth. This model can replicate the above stylized facts for reasonable annual GDP growth rates. In our simulations, inequality over the growth episode is characterized by an inverted U-shaped curve.

The endogenous growth literature of the past decade and a half has focused various endogenous mechanisms. Romer (1986) focused attention on aggregate increasing returns-to-scale. Lucas (1988) & Young (1993) explored learning-by-doing in the production process. Papers by Segerstrom et al (1990), Grossman & Helpman (1991), and Aghion & Howitt (1992) looked Schumpeterian incentives in R&D. These are but a few of the engines of growth that have been explored in this large and growing body of work.

Growth is not new, of course. It was one of the main focuses of Adam Smith's *Wealth of Nations*. Indeed, economic growth inspired the work of most of the classical economists. Two of Smith’s famous concepts are that "extent of the market" drives growth opportunities, and that wealth is created by specialization and exchange. Modern researchers are certainly aware of Smith and the intuitive foundation of many of today's growth models is that growth is driven in by extent of the market in one fashion or another.

Less attention has been paid to the second concept, however - at least in economic growth. This could be because gains from specialization are usually bounded while other engines of growth are not. Since the historic growth experience is long-lived it is appealing to work with models that imply unbounded growth. Still, specialization and exchange are important components of observed economic development. The movement from individual autarky to perfect specialization is fundamentally a change in levels and not rates of growth and this imposes limits on growth. The transition need not be instantaneous, however. As long as specialization does not proceed too rapidly, it is a valid candidate for explaining at least part of observed growth. Any model using specialization as an engine of growth, therefore, must incorporate a reasonable impediment to specialization, which dissipates slowly over time.

The goal of this paper is to create an endogenous model of transitional growth that generates an evolving division of production between households and firms. This is not the first paper to propose a model where specialization plays an important role in growth. Locay (1991) examines a model with an evolving division of production activities between the household and marketplace; his model is driven by scale economies in production and relies upon monitoring costs to provide home production with the advantage at small scales of output. In Locay's model the evolution of markets arises from an exogenous increase in population growth (and, therefore, increased factor supplies and demands).

Yang & Borland (1991) also develop a model which generates growth via specialization. They use identical agents and a large number of final goods. Growth is driven by both learning-by-doing and increasing returns to scale. The economy grows and the market expands as long as two key parameters are neither too small nor too large.

In contrast to these two approaches, this paper offers a model in which aggregate economies of scale arise from the progressive specialization of inherently differentiated labor within a chain of production. Market evolution is limited at each point in time by the presence of interfirm transaction/transportation costs that rise with the increasing thinness of markets in later stages of production – possibly due to greater product differentiation. The evolution of markets arises from declining transaction costs via learning-by-doing spillovers that permit increased aggregate labor productivity through more complete specialization of existing factors according to comparative advantage. Hence, while learning-by-doing drives the dynamics of our model, it does so indirectly through transactions costs and not by direct increases in technical know-how.

We believe that technical progress and the engines of the papers cited above undoubtedly explain some or most of observed economic growth. However, without de-emphasizing these contributions, we wish to explore Adam Smith's simple, yet elegant notions of specialization and extent of the market in the context of aggregate economic growth.

2. The Model

2.1 Preliminaries

We assume a continuum of goods on the unit interval. Consumers have preferences which depend only on consumption of the final good. We also assume that they are each endowed with one unit of labor which they supply inelastically to the labor market since only consumption enters utility. Because goods are not storable and there is no capital, the dynamic problem which consumers solve can be viewed as a series of static problems. Consumers are differentiated by type which is also a number on the unit interval. Individuals of type *i* will be more productive in producing good *i* than all other goods. We assume that individuals are uniformly distributed across all types with enough individuals of each type to ensure competitive markets.

Production in this economy consists of firms or individuals obtaining goods from the immediately previous stage of production and applying labor to these goods to produce value-added. The outcome is goods which can then be passed on to the next stage of production. Production of an arbitrary good, *i*, is assumed to be Leontieff in production labor and materials (goods from the previous stage). The labor portion of production is linear with labor productivity varying by the type of individual doing the work (indexed by $0 \le k \le 1$) and by the production environment (indexed by $s=h,m$; h=home & m=market). For market production, some labor will also be used to transport goods to a location where production of the next stage good occurs. Home production is assumed to occur all in the same place and hence has no transaction costs. The production function for good *i* is

$$Y(i) = Min\left[\frac{L_{ks}(i)}{a_{ks}}, \frac{N_{ks}(i)}{c_{ks}(i)}, M(i)\right], \; a_{ks} = \begin{cases} a_h & \text{if } k \neq i \;\&\; s = h \\ b_h & \text{if } k = i \;\&\; s = h \\ a_m & \text{if } k \neq i \;\&\; s = m \\ b_m & \text{if } k = i \;\&\; s = m \end{cases} \tag{2.1}$$

Where:

$Y(i)$ is the output of production at stage *i*,

$L_{ks}(i)$ is labor of type k in environment s used to produce good i,

a_{ks} is the unit labor requirement in goods production for labor of type k in environment s,

$N_{ks}(i)$ is labor of type k in environment s used to perform transaction services for good I,

$c_{ks}(i)$ is the unit labor requirement in transaction services of good i for labor of type k in environment s, and

$M(i)$ is the materials passed to the immediately next stage from point i.

We assume that $a_h > a_m$, $b_h > b_m$, $a_h > b_h$, $a_m > b_m$, which guarantees that individuals are more productive in their good of specialization regardless of the structure of production. It also guarantees that they are more productive in market production than in home production.[1]

We also assume that

$$c_{ks}(i) = \begin{cases} 0 & \text{if } s = h \\ c_k \tau(i) & \text{if } s = m \end{cases}$$

which gives zero transaction costs for goods produced within the household and positive transaction costs if the goods are produced in the market.

2.2 Autarky Production

In autarky each individual takes free raw materials and passes them through all stages, adding value via household labor inputs at each stage, to final production of the final good. Each individual must choose the optimal level of labor for each stage of production. For simplicity we assume that there is no time element involved in the production and passing of goods from one stage to the next. Rather, production at all levels occurs within the same time period. Since we are focusing on growth over long periods of time we feel this is a reasonable simplification.

Given the nature of production and the constraint that $Y(i) \geq M(i)$ (i.e. the materials passed to the next stage must be less than or equal to production), we can write the production function for final goods for an individual of type k as:

$$Y(1) = Min[Y(i)] = Min[\tfrac{L_{kh}(i)}{a_{kh}}];\ 0 \leq i < 1 \tag{2.2}$$

The optimal choice of outputs for each level of production is described by $Y(i) = Y_a$. This gives the following labor constraint:

$$1 = \int_0^k Y_a a_h di + Y_a b_h di + \int_k^1 Y_a a_h di = Y_a a_h \tag{2.3}$$

Which gives final production & consumption of $Y_a = 1/a_h$. Note Y_a does not depend on the value of k, so all individuals have identical final consumption in autarky.

[1] We motivate this by assuming there is an optimal firm size which is larger than the household.

2.3 Market Production

If individuals engage in household production they can costlessly move materials from one stage of production to the next. However, we assume that if firms engage in production they must pay a transaction cost of $c_k\tau(i)di$ per unit whenever goods are moved from one stage to the next. This gives the following competitive pricing formula:

$$\dot{P}(i) = c_k\tau(i)w_k di + a_{km}w_k di \tag{2.4}$$

$\dot{P}(i)$ shows the increase in the price of goods moving up the chain of production by a small increment in the neighborhood of i, w_k is the wage paid per unit of labor to type k individuals - w_u for unskilled workers and w_s for skilled workers.

Since the output of skilled and unskilled workers are indistinguishable, it must be that $b_m w_s = a_m w_u$. Normalizing so that $w_u=1$ gives $w_s=a_m/b_m$. It also gives $a_{km}w_k=a_m$ for all k.

We assume that $P(0)=0$ since initial materials are free.

We also assume that

$$c_k = \begin{cases} a_m & \text{if } k = i \quad \text{(worker is skilled)} \\ b_m & \text{otherwise} \quad \text{(worker is unskilled)} \end{cases}$$

which implies that skilled workers have an absolute advantage over unskilled workers in the production of goods and in providing transaction services, but they have no comparative advantage.

Integrating eq. (2.4) gives the price of good i, $P(i)$:

$$P(i) = \int [a_m + a_m\tau(i)]dj = a_m[i + T(i)]; \; T(i) \equiv \int_0^i \tau(j)dj \tag{2.5}$$

Now consider the case where markets exist for all stages between 0 and J. In this case an individual could allocate some labor to the market, work for a firm and earn wages which he could use to purchase good J. He must then take good J and transform it via household production into the final good. The individual who does this faces the following budget constraint:

$$P(J)M(J) = w_k\left[1 - \int_J^1 L_{kh}(i)di\right] \tag{2.6}$$

w_k is the wage the individual receives from the labor market and $L_{kh}(i)$ is labor devoted to household production at stage *i*. If the individual is of type $i \leq J$ he will be skilled in producing a good which is produced by firms and sold in the market. Since $w_s = a_m/b_m > w_u$ he will devote all of his market labor to this good or to its transaction services. Individuals who have type $k>J$ will be unskilled and earn $w_u=1$. If they participate in market production they will be indifferent between producing any of the 0 to *J* market goods and supplying transaction services. This gives:

$$w_k = \begin{cases} a_m / b_m & \text{if } k \leq J \\ 1 & \text{otherwise} \end{cases} \tag{2.7}$$

The individual takes his purchases of $M(J)$ and applies labor to obtain $Y(1)$. As in the autarky case we will have $Y_k(i) = Y_k = C_k$ but for all $i>J$.

2.4 Levels of Market Participation

For an individual of type $k>J$ (unskilled) the labor resource constraint can be written as:

$$(1-J)Y_u a_h + JY_u a_m + Y_u a_m T(J) = 1 \tag{2.8}$$

The first term is labor devoted to household production, the second term is the amount of labor allocated to market production and the third term is the amount allocated to transaction services. The first term is the same for every individual of type $k>J$ the second & third terms are the per worker averages, the exact distribution for a given individual across these two activities is indeterminate.

For an individual of type $k \leq J$ (skilled) the labor resource constraint can be written as:

$$(1-J)Y_s a_h + JY_s b_m + Y_s b_m T(J) = 1 \tag{2.9}$$

The second term is a per worker average with every worker supplying this amount to the production of his specialty good *k*.

We assume that the transaction cost of moving goods from one stage to the next increases with the stage. This can be rationalized by noting that goods at later stages are more differentiated than those at earlier stages. Differentiated goods will have thinner markets and higher transaction costs per unit. However, none of these costs apply to household production.

We make the following assumptions about the transportations costs over goods

- $0 < \tau(i)$ and finite for all i<1
- $\tau(i)$ is continuous & monotonically increasing in i
- $\tau(0) < a_h / b_m - 1$

These assumptions assure that there exist unique levels J_S and J_U greater than zero and less than or equal to one that maximize final consumption.

We define $C_S(J)$ as the level of final consumption for a skilled individual who relies on market production for intermediate goods up to stage J. Since $C_S(J)$ is the same as Y_S in (2.9) we can rewrite it as:

$$C_s(J) = [(1-J)a_h + Jb_m + T(J)b_m]^{-1}$$

$C_S(J)$ will be maximized at the value of J_S defined by $\tau(J_s) = a_h/b_m\text{-}1$.

Similarly, $C_U(J)$ is the level of final consumption for an unskilled individual.

$$C_u(J) = [(1-J)a_h + Ja_m + T(J)a_m]^{-1}$$

$C_U(J)$ will be maximized at the value of J_U defined by $\tau(J_u) = a_h/a_m\text{-}1$.

Since $a_h/b_m > a_h/a_m$ and we have $\tau(J_s) > \tau(J_u)$, with $\tau(i)$ monotonically increasing in i it follows that $J_U < J_S$.

In this economy all individuals do some market production. If an individual is of type $k \leq J_S$ he will work as a skilled worker producing good k & providing transaction services. Since there are equal numbers of all skilled workers they will each work the same amount and produce the same amount of product at each stage. This will yield an amount $C_S(J_S)$ of good J_S which is purchased by these individuals using the wages they have earned. They then take good J_S and convert it into final consumption via household production.

Individuals of type $k>J_S$ will work as unskilled workers producing goods between 0 and J_u and providing transaction services. Like skilled workers, they take their wages and buy goods (in this case good J_u) which they also convert to final consumption via household production.

2.5 GDP & Real Consumption

We define two useful measures of economic activity and welfare. First, gross national product, Y, is the value of all goods and services purchased by households. This measure includes the value of all market production in the economy, but excludes the value of household production. There is a measure of J_S individuals each purchasing an amount of good J_S equal to their final consumption level of $C_S(J_S)$ and valued at price $P(J_S)$. There is also a measure of $1-J_S$ individuals who buy an amount of good J_u equal to $C_u(J_u)$ valued at price $P(J_u)$. Thus, Y can be written as:

$$Y = \frac{J_s a_m [J_s + T(J_s)]}{(1-J_s)a_h + J_s b_m + b_m T(J_s)} + \frac{(1-J_s)a_m[J_u + T(J_u)]}{(1-J_u)a_h + J_u a_m + a_m T(J_u)} \tag{2.10}$$

Second, we define total final consumption, C, as the sum of final consumption goods consumed by all individuals. There are J_S individuals consuming $C_S(J_S)$ and $1-J_S$ consuming $C_u(J_u)$. This is:

$$C = \frac{J_s}{(1-J_s)a_h + J_s b_m + b_m T(J_s)} + \frac{(1-J_s)}{(1-J_u)a_h + J_u a_m + a_m T(J_u)} \tag{2.11}$$

We can construct a measure of income inequality based on either Y or C by using the notion of a Gini coefficient. Since there are only two types of agents this is straightforward.

3. Changes in Transaction Costs over Time

As long as the relationship $\tau(i)$ is constant, the economy exhibits no growth. We now incorporate a form of learning-by-doing that leads to transitional growth. We assume that the transaction cost at stage i falls over time as a function of experience in the market.

We assume there is an upper and lower bound for the transaction cost at stage *i*. With no experience the transaction cost sits at the upper bound $\tau^U(i)$. With sufficient experience the cost is lowered to $\tau^L(i)$.

We make the following assumptions about these two functions:

For $\kappa = U, L$:

- $0 < \tau^{\kappa}(i)$ and finite
- $\tau^{\kappa}(i)$ is continuous & monotonically increasing in *i*
- $\tau^{\kappa}(i) < \infty$ for $i < 1$
- $\tau^{U}(i) > \tau^{L}(i)$ for all *i*
- $\tau^{U}(0) < a_h/b_m - 1$
- $\tau^{L}(0) > a_h/b_m - 1$

We assume that the relationship between the actual cost at time *t*, denoted $\tau(i,t)$ and these bounds is a function of experience base on past market transactions in the good and closely related goods. We also assume that experience is a pure public good. If this were not the case then some individuals might choose to "invest" by overproducing now and making losses in exchange for speeding up learning-by-doing. Thus, the transactions cost associated with good *i* in period *t* is

$$\tau(i,t) = \tau^U(i) - f\{E(i,t)\}[\tau^U(i) - \tau^L(i)] \qquad (3.1)$$

We make the following assumptions:

- $f(0) = 0$
- $f(E)$ is continuous & monotonically increasing in *E*
- $\lim_{E\to\infty} f(E) = 1$

We also assume the there are spillovers of experience in transacting a particular good to the cost of transacting goods sufficiently close to it. In particular, that experience at level *i* is a function of all past market sales of goods i-δ through i+δ. Experience accruing to good *i* as a result of market sales of good *j* time period *t* is

$$e(i, j,t) = w(i - j) y(j,t) \qquad (3.2)$$

where $w(d)$ is the amount of experience accruing to good i from a good distance d away on the unit interval. We assume that goods sufficiently different from i give zero weight in providing market experience for good i and the weights greater the closer goods are to i within the window. The formal assumptions for $w(d)$ are:

- $w(d)$ is continuous & monotonically decreasing in d for $d < \delta$.
- $w(d) = 0$ for $d \geq \delta$
- $W(\delta) \equiv \int_0^{\delta} w(j)dj = 1$

Total new experience accruing to good i in period t is thus,

$$\Delta E(i,t) == \int_{i-\delta}^{i+\delta} w(|\, i - j\,|) y(j,t)dj \qquad (3.3)$$ [2]

where $y(j,t)$ is market sales of good j at date t.[3] These will be

$$y(j,t) = \begin{cases} C_u(J_u(t)) + C_u(J_u(t)) & -\delta < j < J_u(t) \\ C_u(J_u(t)) & J_u(t) \leq j \leq J_s(t) \\ 0 & j > J_s(t) \end{cases} \qquad (3.4)$$

$J_s(t)$ & $J_u(t)$ are the values of J_s & J_u in period t.

Total accrued experience since t=0 associated with good j is:

$$E(i,t) = \sum_{s=0}^{t} \Delta E(i,s) \qquad (3.6)$$

Assumptions about $\tau^U(i)$ and $\tau^L(i)$ guarantee that $0 < \tau(i,t)$ and finite for all i<1 and $\tau(0,t) < a_h / b_m - 1$ since $\tau(i,t)$ is a convex combination of $\tau^U(i)$ and $\tau^L(i)$. The Appendix shows that $\tau(i,t)$ is continuous & monotonically increasing in i and, therefore, has all the properties of $\tau(i)$ listed in section 2. Hence, we can choose $J_S(t)$ and $J_U(t)$ as described there. Figure 1 illustrates the evolution of $\tau(i,t)$ over time.

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[2] This functional form requires market sales for goods between $-\delta$ and zero, which do not exist. We modify the weighting structure for experience on goods 0 through $-\delta$, so that these undefined goods enter the weighting scheme with the same weight as good 0. This is shown in equation (3.5) by the range for j.

[3] Goods $i - j \in [-\delta,0), (1,1+\delta]$ do not exist, but we will define the values used in (3.4) as: $y(i+j,t) = y(0,t)$ for $i + j \in [-\delta,0)$; and $y(j,t) = y(1,t)$ for $i + j \in (1,1+\delta]$. This helps assure that good 0 gets just as much experience as good i if $J_U \geq \delta + i$.

We can now set forth propositions about the behavior of the economy during its period of transitional growth.

Proposition 1: If $\tau^L(0) > a_h/a_m - 1$ *then no unskilled market production will ever occur.*

Proof: Since $J_u(t)=0$ if $\tau(0,t) > a_h/a_m - 1$ and $\tau(0,t) \geq \tau^L(0) > a_h/a_m - 1$ and since $\tau^L(0)$ is monotonically increasing in i we know that $\tau(i,t) > a_h/a_m - 1$. Hence no unskilled production can ever occur.

Proposition 2: Some skilled market production will occur at $t=0$ *and the set of goods so produced will grow but must stop short of some upper bound less than* 1. *If unskilled market production occurs the set of such goods produced will grow over time, but must stop short of some upper bound less than* 1.

Proof: $J_s(0)$ is defined by $\tau^U(J_s(0)) = a_h/b_m - 1$ and $\tau^U(0) < a_h/b_m - 1$, so $J_s(0)>0$ and individuals of types 0 to $J_s(0)$ will provide labor to the market. $\tau(i,t)$ falls over time, but it has a lower bound of $\tau^L(i)$. Since $\tau^L(1) > a_h/b_m - 1$ there is an upper bound on $J_s(t)$ even as t goes to infinity. This upper bound, J_s^*, is defined by $\tau^L(J_s^*) = a_h/b_m - 1$. Proposition 1 shows that unskilled production will eventually occur if $\tau^L(0) < a_h/a_m - 1$. Logic similar to that for skilled goods shows that there is an upper bound of J_u^* as well.

Proposition 3: Our measures of production, Y and C will grow over time, but are also bounded from above.

Proof: Follows from Propositions 1 & 2. The upper bounds are found by substituting J_s^* for J_s and J_u^* for J_u if $\tau^L(0) > a_h/a_m - 1$ or 0 for J_u if $\tau^L(0) < a_h/a_m - 1$ into eqs. (2.10) and (2.11).

4. Simulation Results

To simulate this model we approximate our continuous setup by constructing a discrete grid on i in the range [0,1] divided into N equally sized segments.

In order to check the robustness of our simulation results we need functional forms for $\tau^U(i)$, $\tau^L(i)$, $f(E)$ and $w(d)$, that allow for great flexibility in shape; subject to the assumptions made above.

First we specify functional forms for $\tau^U(i)$ and $\tau^U(i)$ as

$$\tau^U(i) = c(1-\varepsilon_0) + [\tfrac{1}{\mu}c(1+\varepsilon_1) - c(1-\varepsilon_0)]/i^\gamma$$

$$\tau^L(i) = \mu c(1-\varepsilon_0) + [c(1+\varepsilon_1) - \mu c(1-\varepsilon_0)]/i^\gamma$$

where $c \equiv \frac{a_h}{b_m} - 1$, the critical value for market participation by skilled individuals. $\varepsilon_0 > 0$, $\varepsilon_1 > 0$, $\mu < 1$, and $\gamma > 0$ are free parameters. The ε's determine the endpoints. Note that $\tau^U(0) = c(1-\varepsilon_0)$ which implies that there is some skilled market participation in the first period of the simulation. Note also that $\tau^L(1) = c(1+\varepsilon_1)$ which implies that even there is always some household production even for skilled individuals. μ indicates the potential transactions cost savings from experience. A condition necessary for market participation by unskilled individuals is $\mu < (\frac{a_h}{a_m} - 1)/(\frac{a_h}{b_m} - 1)$. γ controls curvature. Figure 2 plots $\tau^L(i)$ for various values of γ.

Next we specify $f(E)$ as

$$f(E) = \frac{Tan^{-k1}(\kappa_2 E - \kappa_1) - Tan^{-k1}(-\kappa_1)}{\frac{\pi}{2} - -Tan^{-k1}(-\kappa_1)} \quad \kappa_2 > 0$$

$\kappa_2 > 0$ is a sensitivity parameter and κ_1 controls the shape of the curve. Figure 3 plots $f(E)$ for various values of κ_1.

Finally, we specify $w(d)$ as

$$w(d) = \frac{\exp\{-\phi_1(\frac{d}{\delta})^{\phi_2}\} - \exp\{-\phi_1\}}{1 - \exp\{-\phi_1\}}; \text{ for } d \le \delta$$

$\phi_1 > 0$ is a truncation parameter that guarantees $w(\delta)=0$ and $w(0)=1$. $\phi_2 > 0$ governs how quickly the weights decline as a function of d. Figure 4 plots $w(d)$ for various values of ϕ_2.

We begin our simulations by using a grid with N=10,000 and running a 1000-period simulation. We set $a_h = 1$, $a_m = .1$, $b_m = .01$ which implies that market production is ten times as efficient as home production and that skilled workers are ten times as productive as unskilled workers. We set $\gamma = 1$, which gives transactions cost functions that are linear in *i*. We also use $\varepsilon_0 = .1$, $\varepsilon_1 = .01$, and $\mu = .009$. We set $\kappa_1 = 0$, which ensures that $f'(E) > 0$ and $f''(E) < 0$, while $\kappa_2 = 20$ is chosen so that all potential benefits from specialization have dissipated by the end of the simulation. We use $\delta = .05$ giving a spillover range that covers 10% of all goods, and set $\phi_1 = 4.5$, $\phi_1 = 2$, so that the weights are a truncated bell curve.

Figures 5 and 6 plot growth rates and inequality measures based on GDP and final consumption over time. Skilled individuals begin in period 1 by buying good .0009 on the unit interval. Because output of this good is so low and so few individuals are skilled, experience accumulates very slowly. The economy continues in this near-autarky state for over 200 periods. Gini coefficients based on GDP are close to one because all income is earned by the few skilled individuals. In period 273, skilled participation has expanded and market experience has accumulated enough that unskilled individuals begin to participate in the market. Gini coefficients fall and from this point on there is substantial growth. As figure 6 shows, growth of both GDP and final consumption are well above 5% per period. Growth rates gradually drop, while inequality rises at first and then falls. The final steady state is one with market J_s=.9741 and J_u=.0813.

In this simulation we find an inverted U in inequality over time for both GDP and final consumption. That is, as output expands after the entry of the unskilled into the market, inequality initially rises and then falls. The simulation also gives growth rates that are initially very high, but which slowly decline to zero. Despite this, the growth experience is one that lasts for several hundred periods. Starting in period 224 growth rates are continuously above 0.5% for 342 periods. 1.5% is a reasonable ballpark figure of the growth of output per capita in market economies. Allowing that there are other important sources of growth beside specialization, an extended growth episode with one-third the observed rate is encouraging.

The biggest problem with this simulation is that growth rates decay slowly over time. In his discussion of increasing returns to scale, Romer (1986) cites evidence that growth rates have risen since the industrial revolution. We next search over parameter values to see if our model can replicate this.

We conducted several simulations varying the shape and size of the $w(d)$ function, but find that results are not sensitive to these changes. For the remainder of the paper we retain a bell shape for $w(d)$.

Once growth becomes established, it proceeds rapidly at first and slowly later on. This is due to the interaction between the transactions cost function and the experience function, $f(E)$. As growth proceeds the volume of goods sold in the market expands for all markets. This, in turn, leads to more rapid accumulation of experience. When experience rises faster, transactions costs fall faster, which causes growth to proceed even more rapidly.

One way to slow this process would be to have convex transactions cost functions. As experience rises, its rapid accumulation is offset by the need to have even larger drops in cost for later stages of production. The right balance between these two effects could lead to roughly constant growth over some range. We implement this strategy using simulations with various values of γ greater than one. We find no cases, however, where this leads to non-declining growth rates.

A second way to slow the process would be to assume that more experience is need for later stage markets. This could also be rationalized by greater product differentiation at later stages. As experience is spread over more markets, more is needed for the same amount of cost reduction. We simulate this by assuming that experience accumulated for a given amount of market sales is proportional to the inverse of the lower cost function, $\tau^L(i)$. We find, as above, that this has little effect even in conjunction with convex transactions costs.

A third way to proceed is to make the experience function non-concave. As figure 3 shows, values of κ_1 greater than zero lead to experience functions with inflection points. Experience initially has a small impact at the margin, the marginal impact then rises and finally falls again to zero in the limit. When values of κ_1 become very large, $f(E)$ approximates a threshold function. With a small value for δ this could lead to near constant growth. Each stage must have sufficient experience spillover from lower stages before the threshold is crossed, so extent of the market would expand roughly the same amount each period. When we simulate using a value of $\kappa_1 = 40$ and $\gamma = 2$, we get the time paths plotted in figures 7 and 8. Starting in period 102, when unskilled individuals begin to participate in the market, the economy grows at more the 0.5% for 412 periods. In addition, there is a period of roughly 100 periods where growth rates actually rise.

By further fine-tuning it is possible to for the model to generate periods of roughly constant growth that lasts for extended periods of time. Figures 7 and 8 illustrate a simulation with very convex transactions costs ($\gamma = 10$), an experience threshold for cost reduction ($\kappa_1 = 100$), and a decreasing impact of experience on costs for later stages (inversely proportional to the square-root of $\tau^L(i)$. In this simulation the unskilled begin market participation in period 40 and thereafter, growth of GDP remains above 0.5% for 471 periods.

5. Summary & Conclusions

This paper considers a model of transitional growth driven by the movement from individual autarky to specialization and exchange in the market. Abstracting from other sources of growth that are undoubtedly important, we show that this kind of transitional growth is capable of explaining a substantial portion of the historic growth experience. More specifically, we show that growth of more than 0.5%, or roughly one-third observed growth, can be sustained by this process for several centuries. In addition, we show that when such growth occurs inequality initially rises as the economy begins to grow, but falls as the growth process matures.

Our model is quite simple along many dimensions. There is only one factor of production and both capital accumulation and technical progress are absent. It may be that incorporating specialization into a more realistic growth model would yield a better fit with the data than either model does alone. Growth in our model is driven by spillovers of market experience which lower the costs of buying and selling goods nearby in the production chain. It may be that incorporating exogenous technical progress or endogenous investment in transportation technology would be even more fruitful ways of driving specialization. We leave this for future research.

Appendix

Lemma: $\tau(i,t)$ *is continuous & monotonically increasing in i.*

Proof: Recall equations (3.4) & (3.5)

$$\Delta E(i,t) = \int_{i-\delta}^{i+\delta} w(|i-j|)y(j,t)dj$$

$$y(j,t) = \begin{cases} C_u(J_u(t)) + C_u(J_u(t)) & -\delta < j < J_u(t) \\ C_u(J_u(t)) & J_u(t) \le j \le J_s(t) \\ 0 & j > J_s(t) \end{cases}$$

Combining these two and temporarily suppressing the time variable gives

$$\Delta E(i,t) = \begin{cases} 2[C_u(J_u) + C_s(J_s)] & \text{for } -\delta \le i < J_u - \delta \\ [1+W(J_u - i)][C_u(J_u) + C_s(J_s)] + [1-W(J_u - i)]C_s(J_s) & \text{for } J_u - \delta \le i < J_u \\ [1-W(i - J_u)][C_u(J_u) + C_s(J_s)] + [1+W(i - J_u)]C_s(J_s) & \text{for } J_u \le i < J_u + \delta \\ 2C_s(J_s) & \text{for } J_u + \delta \le i < J_s - \delta \\ [1+W(J_s - i)]C_s(J_s) & \text{for } J_s - \delta \le i < J_s \\ [1-W(i - J_s)]C_s(J_s) & \text{for } J_s \le i < J_s + \delta \\ 0 & \text{for } J_s + \delta \le i \le 1 \end{cases}$$

which is continuous and monotonically non-increasing in *i*.

Equation (3.6) along with $E(i,0) = 0 \; \forall i$ guarantees that $E(i,t)$ is continuous and monotonically non-increasing in *i* for all *t*.

Our assumptions about *f*(*E*) ensure that $\alpha(i,t)$ is continuous and monotonically non-increasing in *i* for all *t*.

Since $\tau^U(i)$ and $\tau^L(i)$ are continuous and monotonically increasing in *i*, equation (3.7) is sufficient to ensure that $\tau(i,t)$ is continuous and monotonically increasing in *i* as well.

Figure 1
Evolution of Transaction Costs & Market Participation over Time

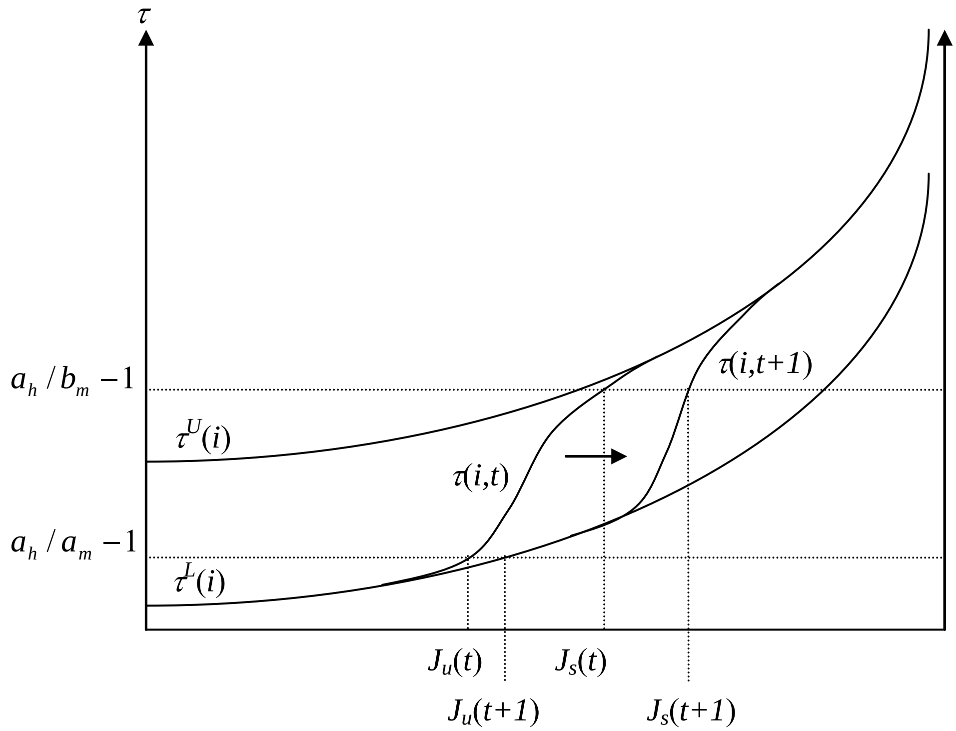


Figure 2

$\tau^{L}(i)$ for various values of γ

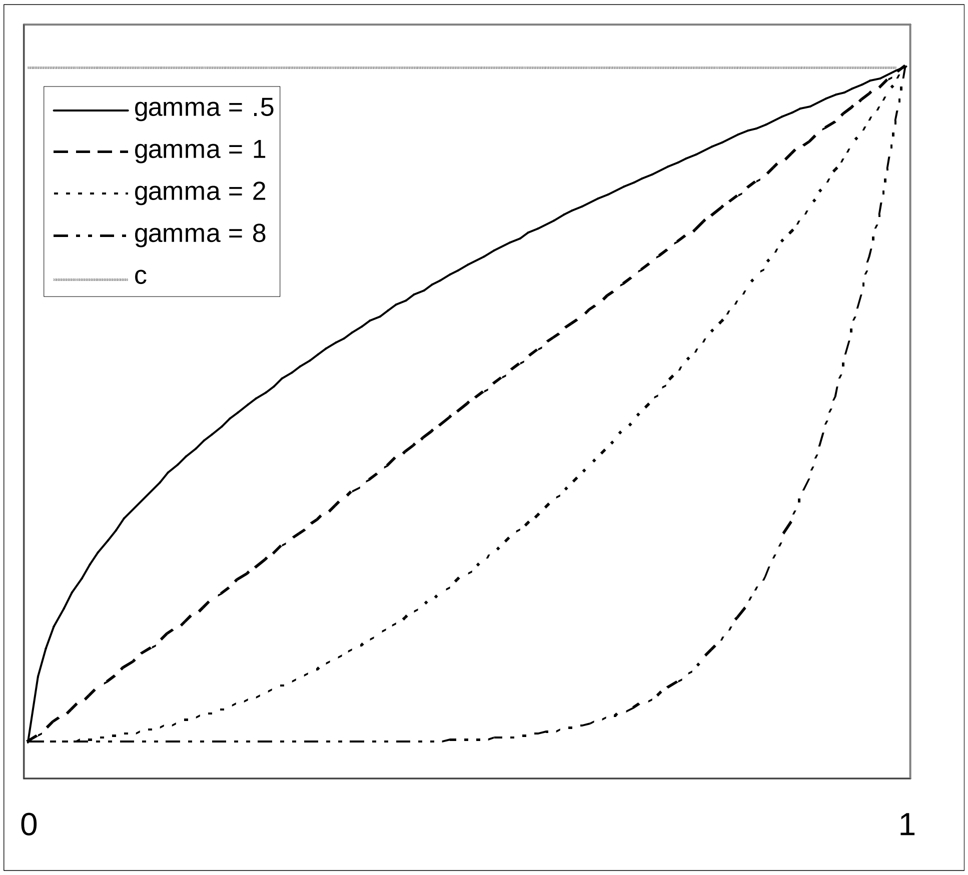


Figure 3

$f(E)$ for various values of κ_1

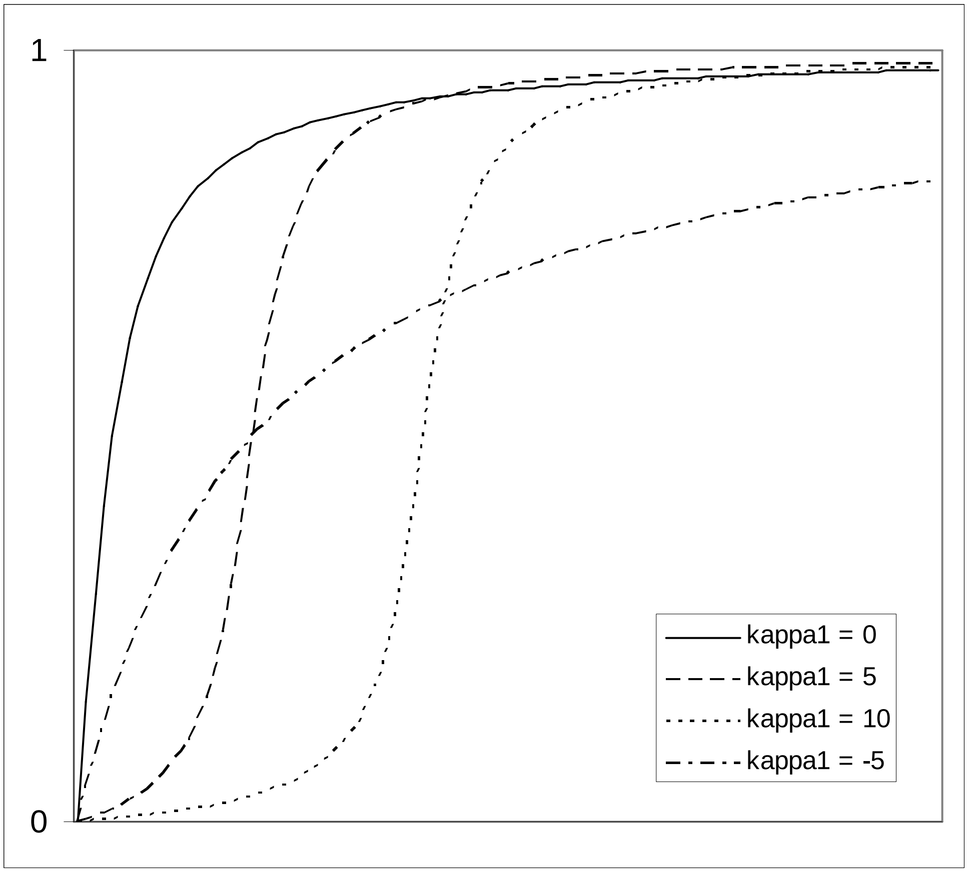


Figure 4

$w(d)$ for various values of ϕ_2

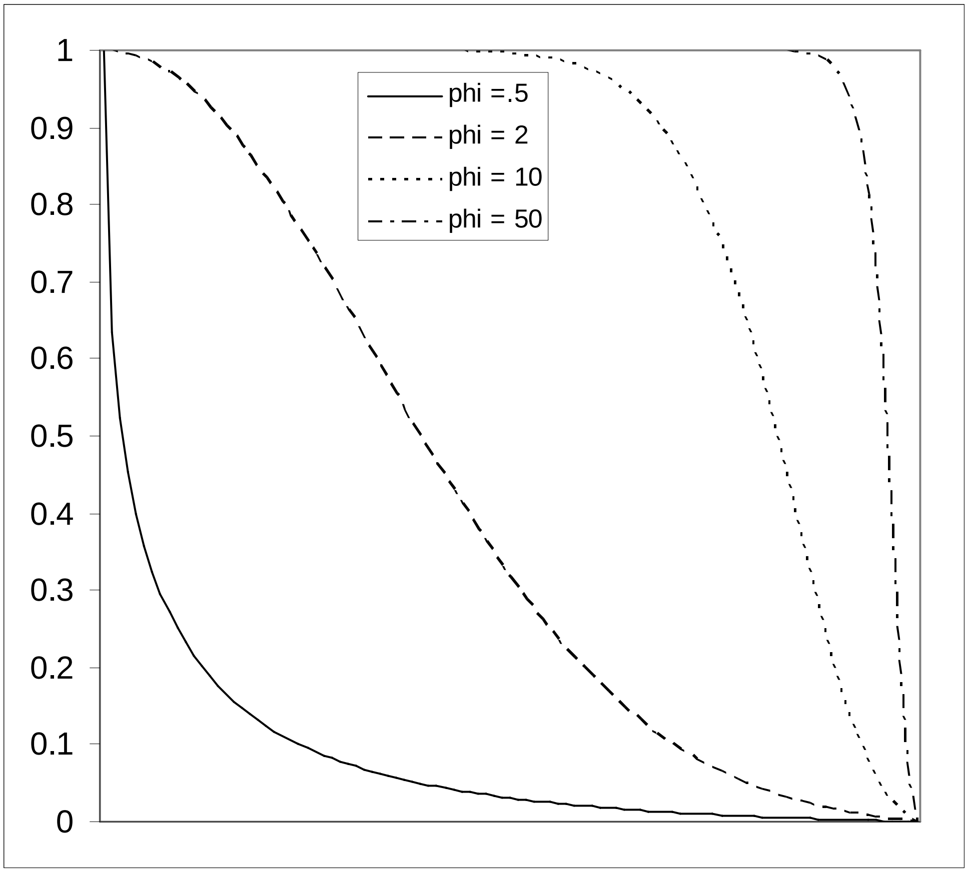


Figure 5

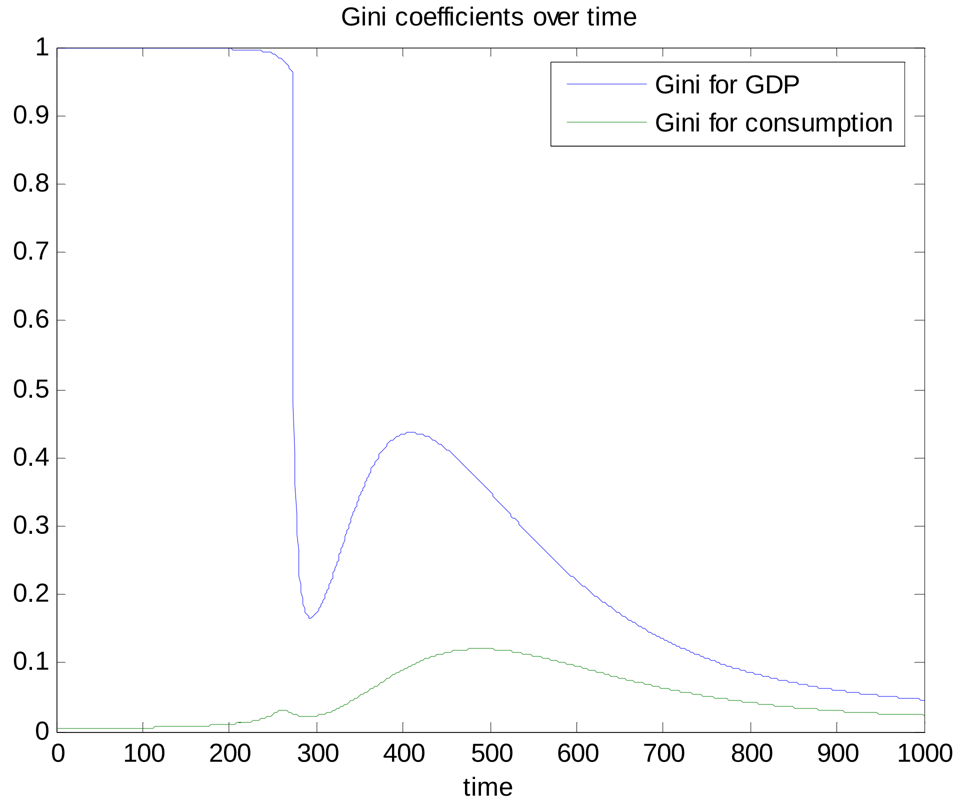

Gini coefficients over time
Gini for GDP
Gini for consumption
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
100
200
300
400
500
600
700
800
900
1000
time

Figure 6

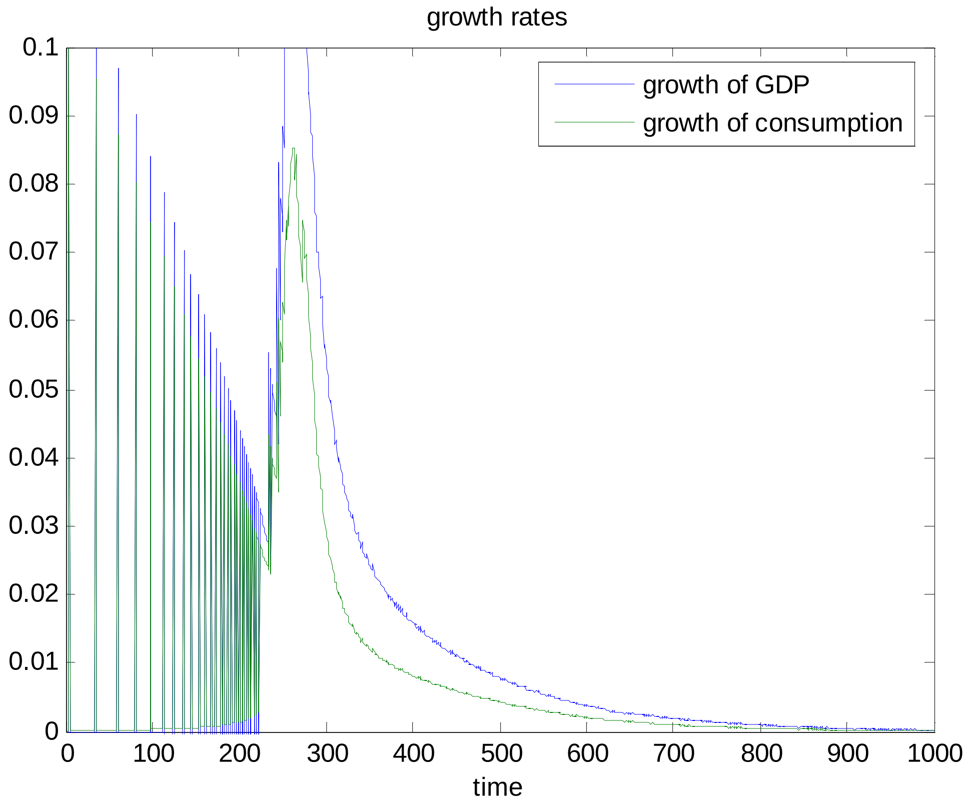

growth rates
growth of GDP
growth of consumption
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
100
200
300
400
500
600
700
800
900
1000
time

Figure 7

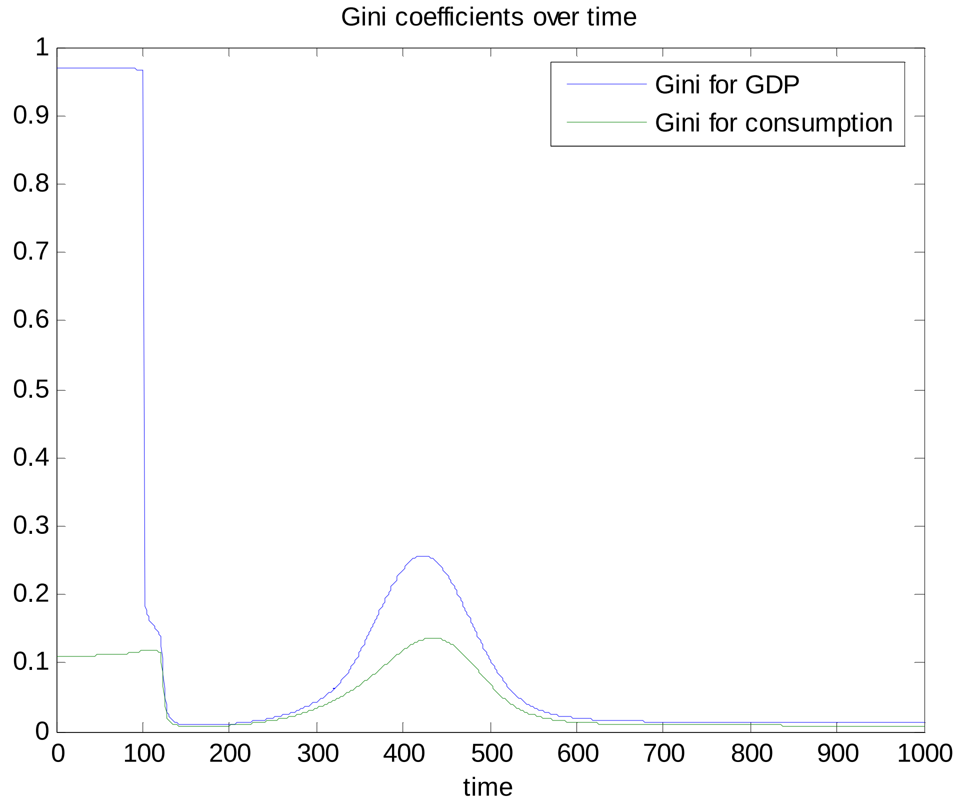
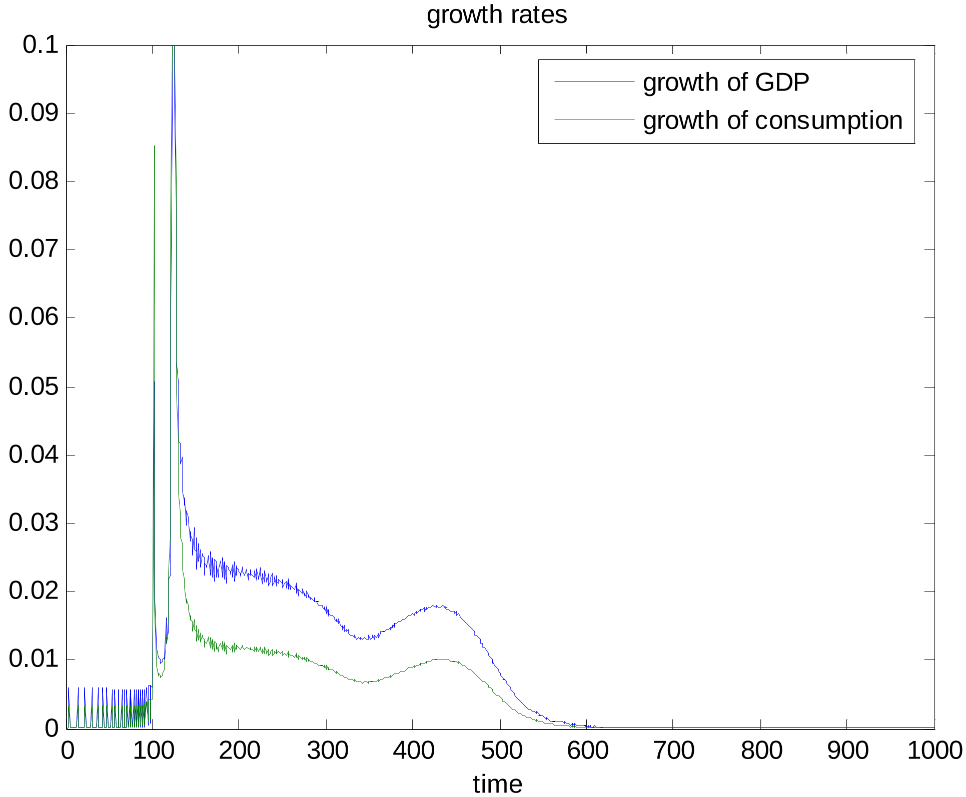
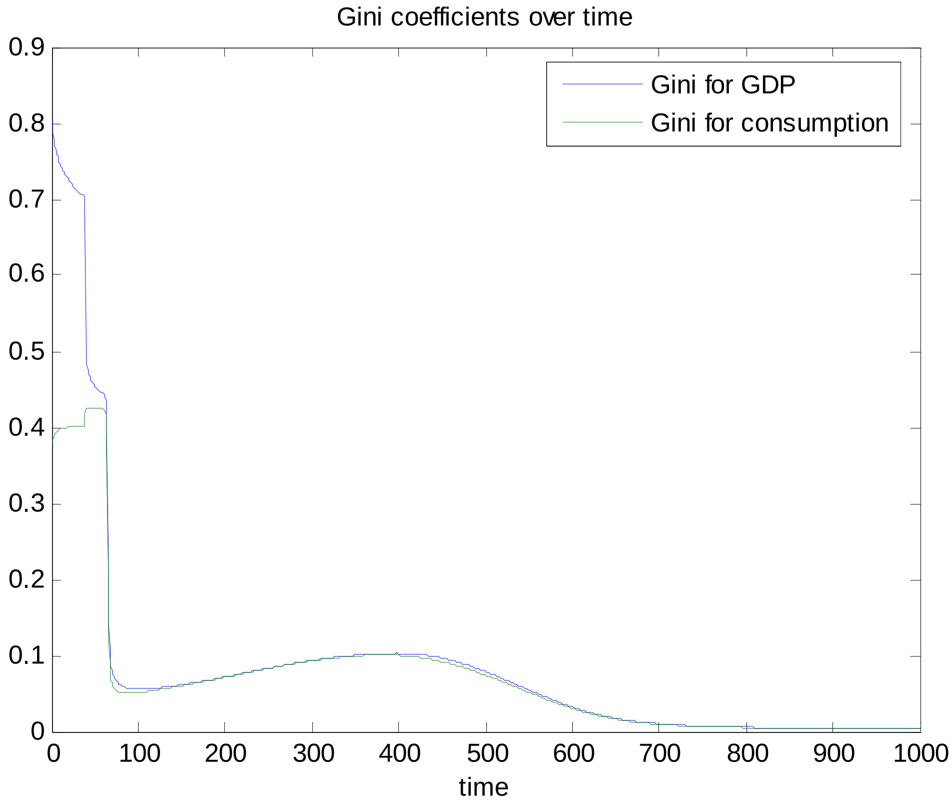

Gini coefficients over time
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
100
200
300
400
500
600
700
800
900
1000
time
Gini for GDP
Gini for consumption

Figure 8



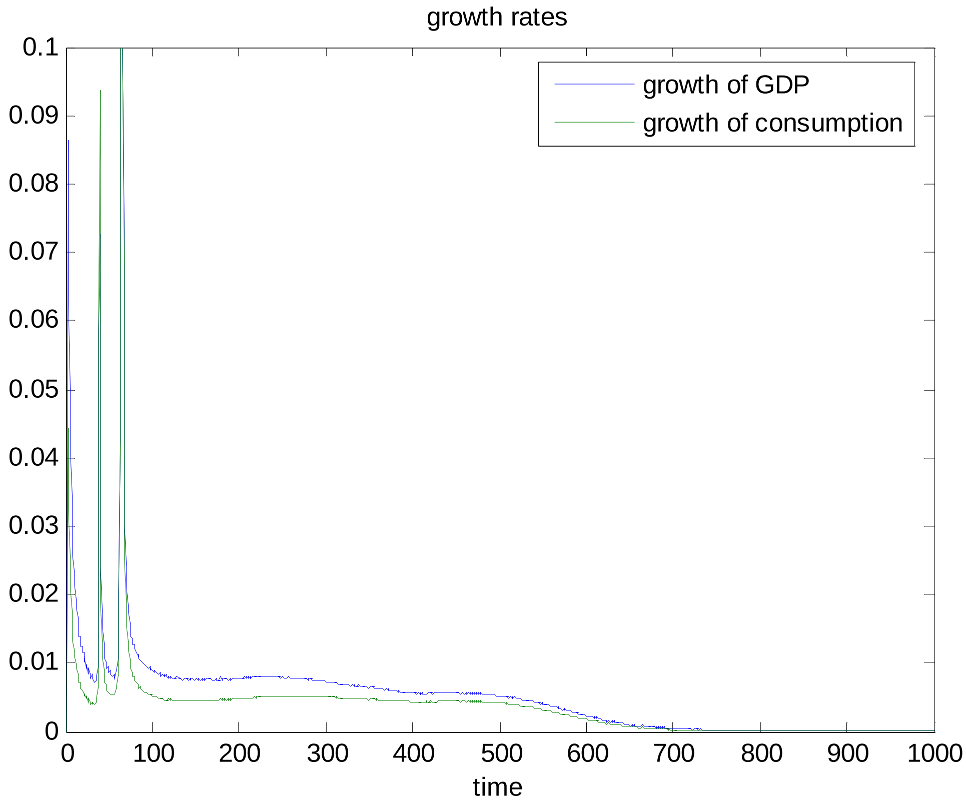
growth rates
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
growth of GDP
growth of consumption
0
100
200
300
400
500
600
700
800
900
1000
time

Figure 9



Gini coefficients over time
Gini for GDP
Gini for consumption
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
100
200
300
400
500
600
700
800
900
1000
time

Figure 10


growth rates
growth of GDP
growth of consumption
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
100
200
300
400
500
600
700
800
900
1000
time

References

Aghion, P. and P. Howitt (1992), "A Model of Growth through Creative Destruction," Econometrica, vol. 60, no. 2 pp. 323-51.

Arrow, Kenneth J. (1962) "The Economic Implications of Learning by Doing." Review of Economic Studies vol. 29 pp. 155-173.

Devereux, John & Luis Locay. (1992) "Specialization, Household Production. and the Measurement of Economic Growth." American Economic Review vol. 82 no. 2 pp. 399-403.

Grossman, G.M. and E. Helpman (1991), "Quality Ladders in the Theory of Growth," *Review of Economic Studies*, vol. 63, pp. 43-61.

Locay, Luis. (1990) "Economic Development and the Division of Production between Households and Markets." Journal of Political Economy vol. 98 no. 5 pp. 965-982.

Lucas, Robert E. , Jr. (1988) "On the Mechanics of Economic Development." Journal of Monetary Economics vol. 22 no. 1 pp. 3-42.

Romer, Paul M. (1986) "Increasing Returns and Long-Run Growth." Journal of Political Economy vol. 94 no. 5 pp. 1002-1037.

Solow, Robert M. (1957) "Technical Change and the Aggregate Production Function." Review of Economics and Statistics vol. 39 no. 3 pp. 312-320.

Segerstrom, P.S., T.C.A. Anant, and E. Dinopoulos (1990), "A Schumpeterian Model of the Product Life Cycle," *American Economic Review*, vol. 80, pp.1077-1091.

Summers, Robert & Alan Heston. (1991) "The Penn World Table (Mark 5): An Expanded Set of International Comparisons: 1950-1988" Quarterly Journal of Economics vol. 106 no. 425 pp. 327-368.

Yang, Xiakai, and Borland Jeff. (1991) "A Microeconomic Mechanism for Economic Growth." Journal of Political Economy vol. 99 no. 3 pp. 460-482.

Young, Alwyn. (1993) "Invention and Bounded Learning by Doing." Journal of Political Economy vol. 101 no. 3 pp. 443-473.