Brigham Young University
Economics 581 – Advanced Macroeconomics
Fall Semester 2006

Final Exam key

This exam is open book and open notes. You may attach (clearly written, easily legible) notes to this exam to show your work if you wish.

1. (40 points) Using the RBC model from the final study sheet and keeping the same notation established there, consider the following modification. Suppose the government taxes economic activity and uses the revenue to buy government goods & services. Assume that there is no preference shock (*y* in the notes). Treat government tax rates as constants.

Assume that household utility depends on government goods & services as follows:

$$u(C,G) = \tfrac{1}{1-\sigma}(C^{\gamma}G^{1-\gamma})^{1-\sigma}$$

Further assume that government revenue is raised entirely through a flat tax on household income with a balanced budget, as follows:

$$G = \tau(wL + rK)$$

Write down the household's problem in the form of a Bellman equation.

$$V(K,\mathbf{\Omega}) = \underset{K'}{Max}\left[\tfrac{1}{1-\sigma}(C^{\gamma}G^{1-\gamma})^{1-\sigma} + \beta E\{V(K',\mathbf{\Omega}')\}\right]$$
$$C = (1-\tau)(wL + rK) + (1-\delta)K - K'$$

Solve this problem to get the relevant Euler equation(s).

First-order condition on *K'*: $\gamma C^{\gamma(1-\sigma)-1}G^{(1-\gamma)(1-\sigma)}(-1) + \beta E\{V_K(K',\mathbf{\Omega}')\} = 0$

Envelope condition is: $V_K(K,\Omega) = \gamma C^{\gamma(1-\sigma)-1}G^{(1-\gamma)(1-\sigma)}[1-\delta+(1-\tau)r]$

(This is more subtle than I realized and made the problem appear much harder than it really is. The household is small and cannot influence the size of *G* through its choice of *K*. It therefore views *G* as a constant, just as it does *w* & *r*. In general equilibrium, of course, *G* does depend on *K*, both directly and through *K*'s effects on *r* and *w*)

Euler equation is: $$1 = \beta E\left\{\left(\frac{C'}{C}\right)^{\gamma(1-\sigma)-1}\left(\frac{G'}{G}\right)^{(1-\gamma)(1-\sigma)}[1-\delta+(1-\tau)r']\right\}$$

List the exogenous and endogenous state variables.

Exogenous is *z*, endogenous is *K*, all else are not state variables.

Assume that z follows a stationary AR process, i.e. $0 < \rho < 1$. Write down a stationary version of this model below. Explain how you rendered the non-stationary variables stationary.

We induce stationarity by defining the growing variables (K, Y, C, w, G) by $A \equiv e^{gt}$.

$$z' = \rho z + \varepsilon_z' ;\ \varepsilon_z' \sim N(0, \sigma_z^2) \tag{1}$$
$$\hat{Y} = \hat{K}^{\alpha} (e^z L)^{1-\alpha} \tag{2}$$
$$\hat{w} L = (1-\alpha)\hat{Y} \tag{3}$$
$$r\hat{K} = \alpha \hat{Y} \tag{4}$$
$$\hat{G} = \tau(\hat{w}L + r\hat{K}) \tag{5}$$
$$\hat{C} = (1-\tau)(\hat{w}L + r\hat{K}) + (1-\delta)\hat{K} - (1+g)\hat{K}' \tag{6}$$
$$1 = \beta E\left\{ \left(\frac{\hat{C}'}{\hat{C}}\right)^{\gamma(1-\sigma)-1} \left(\frac{\hat{G}'}{\hat{G}}\right)^{(1-\gamma)(1-\sigma)} (1+g)^{-\sigma}[1-\delta+(1-\tau)r'] \right\} \tag{7}$$

Use this system to find the steady state value of K as a function of the model's parameters.

The steady state is found by replacing the time-dependent values of variables with their steady state values and solving the system of equations:

$$\bar{z} = 0 \tag{1}$$
$$\bar{Y} = \bar{K}^{\alpha} L^{1-\alpha} \tag{2}$$
$$\bar{w}L = (1-\alpha)\bar{Y} \tag{3}$$
$$\bar{r}\bar{K} = \alpha\bar{Y} \tag{4}$$
$$\bar{G} = \tau(\bar{w}L + \bar{r}\bar{K}) \tag{5}$$
$$\bar{C} = (1-\tau)(\bar{w}L + \bar{r}\bar{K}) - (g-\delta)\bar{K} \tag{6}$$
$$1 = \beta(1+g)^{-\sigma}[1-\delta+(1-\tau)\bar{r}] \tag{7}$$

(7) gives $\bar{r} = \dfrac{\beta^{-1}(1+g)^{\sigma} - 1 + \delta}{1-\tau}$ (7')

(2) & (4) give:

$$\bar{K} = \left(\frac{\alpha}{\bar{r}}\right)^{\frac{1}{1-\alpha}} L \tag{4'}$$

(7') into this gives:

$$\bar{K} = \left(\frac{\alpha(1-\tau)}{\beta^{-1}(1+g)^{\sigma} + \delta - 1}\right)^{\frac{1}{1-\alpha}} L \tag{4''}$$

2. (30 points) Suppose the government imposed a consumption tax instead.

$$G = \tau[wL + (1-\delta+r)K - K']$$

Write the transformed system below.

$$z' = \rho z + \varepsilon_z'; \ \varepsilon_z' \sim N(0,\sigma_z^2) \tag{1}$$
$$\hat{Y} = \hat{K}^{\alpha}(e^z L)^{1-\alpha} \tag{2}$$
$$\hat{w}L = (1-\alpha)\hat{Y} \tag{3}$$
$$r\hat{K} = \alpha\hat{Y} \tag{4}$$
$$\hat{G} = \tau[\hat{w}L + (1-\delta+r)\hat{K} - (1+g+\Delta z')\hat{K}'] \tag{5}$$
$$\hat{C} = (1-\tau)[\hat{w}L + (1-\delta+r)\hat{K} - (1+g)\hat{K}'] \tag{6}$$
$$1 = \beta E\left\{\left(\frac{\hat{C}'}{\hat{C}}\right)^{\gamma(1-\sigma)-1}\left(\frac{\hat{G}'}{\hat{G}}\right)^{(1-\gamma)(1-\sigma)}(1+g)^{-\sigma}[(1-\tau)(1-\delta+r')]\right\} \tag{7}$$

Proceed as in question 1 to find the steady state value of K. Is it higher or lower than in question 1?

The steady state is given by:

$$\bar{z} = 0 \tag{1}$$
$$\bar{Y} = \bar{K}^{\alpha}L^{1-\alpha} \tag{2}$$
$$\bar{w}L = (1-\alpha)\bar{Y} \tag{3}$$
$$\bar{r}\bar{K} = \alpha\bar{Y} \tag{4}$$
$$\bar{G} = \tau[\bar{w}L + (\bar{r}-g-\delta)\bar{K}] \tag{5}$$
$$\hat{C} = (1-\tau)[\bar{w}L(\bar{r}-g-\delta)\bar{K}] \tag{6}$$
$$1 = \beta(1+g)^{-\sigma}(1-\tau)(1-\delta+r') \tag{7}$$

(7) gives $\bar{r} = \dfrac{\beta^{-1}(1+g)^{\sigma}}{1-\tau} - 1 + \delta$ (7')

(2) & (4) give:

$$\bar{K} = \left(\frac{\alpha}{\bar{r}}\right)^{\frac{1}{1-\alpha}} L \tag{4'}$$

(7') into this gives:

$$\bar{K} = \left(\frac{\alpha(1-\tau)}{\beta^{-1}(1+g)^{\sigma} + (\delta-1)(1-\tau)}\right)^{\frac{1}{1-\alpha}} L \tag{4''}$$

So $\bar{K}$ is bigger in this case than in question 1.

3. (30 points) Going back to problem 1, suppose that government tax policy is not constant, but follows a stochastic process with at least some unpredictability. In particular suppose the income tax rate follows a first-order autoregressive process as follows:

$$\tau = \bar{\tau} e^{x};\ x' = \phi x + \varepsilon_x';\ \varepsilon_x' \sim N(0, \sigma_\eta^2)$$

List the exogenous and endogenous state variables.

Exogenous are *z* & *x*, endogenous is *K*, all else are not state variables

Write down a stationary version of this model below.

$$z' = \rho z + \varepsilon_z';\ \varepsilon_z' \sim N(0, \sigma_z^2) \quad (1)$$

$$x' = \phi x + \varepsilon_x';\ \varepsilon_x' \sim N(0, \sigma_\eta^2) \quad (2)$$

$$\hat{Y} = \hat{K}^{\alpha}(e^{z}L)^{1-\alpha} \quad (3)$$

$$\hat{w}L = (1-\alpha)\hat{Y} \quad (4)$$

$$r\hat{K} = \alpha\hat{Y} \quad (5)$$

$$\hat{G} = \bar{\tau}e^{x}(\hat{w}L + r\hat{K}) \quad (5)$$

$$\hat{C} = (1-\bar{\tau}e^{x})(\hat{w}L + r\hat{K}) + (1-\delta)\hat{K} - (1+g)\hat{K}' \quad (6)$$

$$1 = \beta E\left\{\left(\frac{\hat{C}'}{\hat{C}}\right)^{\gamma(1-\sigma)-1}\left(\frac{\hat{G}'}{\hat{G}}\right)^{(1-\gamma)(1-\sigma)}(1+g)^{-\sigma}[1-\delta+(1-\bar{\tau}e^{x'})r']\right\} \quad (7)$$

How does solving for the approximate linear decision rule(s) in this model differ from solving for them in question 1?

In this case there are two exogenous variables and we would have to find approximate linear decision rules where the deviation of $\hat{K}$ from its steady state value is a function of the deviation of *x* from its steady state value of zero as well as deviations of *z* away from zero as we have in question 1. This means taking derivatives of equation (7) with respect to *x* and *z* both.

Looking at this system, but without solving explicitly, how would its steady state compare with that in question 1?

The steady state will be the same only we replace τ from question 1 with $\bar{\tau}$.