Brigham Young University Department of Economics

Economics 459 - International Monetary Theory

Useful Properties of Expected Values, Variances & Covariances

The expected value, or mean, of a random variable, say *x*, is defined by, $E\{x\} = \int x\ f(x)\,dx$, where *f*(*x*) is the probability density function.

The variance of a random variables is defined as, $Var\{x\} = E\{(x - E\{x\})^2\}$, and the square root of this is called the standard deviation. The standard deviation can be interpreted as how far from the mean the variable is on average. Both the standard deviation and the variance must be positive.

The covariance of two random variables, say x_1 & x_2, is defined as, $Cov\{x_1, x_2\} = E\{(x_1 - E\{x_1\})(x_2 - E\{x_2\})\}$. Note that if x_1 tends to be above its mean when x_2 is above its mean also, this will evaluate to a positive value. If x_1 tends to be below its means when x_2 is above its, then the covariance will be negative. Two random variables are said to be "independent" if their covariance is zero. Also note that if $x_2 = x_1$, i.e. we consider the covariance of x_1 with itself, we simply get the formula for variance above. It is also the case that $Cov\{x_1, x_2\} = Cov\{x_2, x_1\}$. Note random variables with high variances will also have high covariances.

To correct for this, there is a statistic related to the covariance, the correlation coefficient, which must take on a value between -1 and 1, and which is defined by:

$$Corr\{x_1, x_2\} = \frac{Cov\{x_1, x_2\}}{\sqrt{Var\{x_1\}Var\{x_2\}}}$$

If *y* is a linear function of two random variables,

$$y = w_1 x_1 + w_2 x_2$$

then the expected value of *y* is:

$$E\{y\} = w_1 E\{x_1\} + w_2 E\{x_2\}$$

and the variance is:

$$Var\{y\} = w_1^2 Var\{x_1\} + w_2^2 Var\{x_2\} + 2 w_1 w_2 Cov\{x_1, x_2\}$$

The covariance of *y* with any other random variable, say *z*, is:

$$Cov\{y, z\} = w_1 Cov\{x_1, z\} + w_2 Cov\{x_2, z\}$$

More generally, if, $y = \sum_i w_i x_i$ then:

$$E\{y\} = \sum_i w_i E\{x_i\}$$

$$Var\{y\} = \sum_i \sum_j w_i w_j Cov\{x_i, x_j\}$$

noting $Cov\{x_i, x_i\} = Var\{x_i\}$ we can also write this as:

$$Var\{y\} = \sum_i w_i^2 Var\{x_i\} + 2\sum_i \sum_{j<i} w_i w_j Cov\{x_i, x_j\}$$

$$Cov\{y, z\} = \sum_i w_i Cov\{x_i, z\}$$

Lastly, if two random variables, y_1 & y_2, are linear functions of other random variables,

$$y_1 = \sum_i w_{1i} x_i \text{ and } y_2 = \sum_i w_{2i} x_i$$

Then the covariance between the two is given by:

$$Cov\{y_1, y_2\} = \sum_i \sum_j w_{1i} w_{2j} Cov(x_i, x_j)$$