Brigham Young University Department of Economics

Economics 459 - International Monetary Theory

Two-period Model of the Current Account

Consumer's Problem

$$\max_{C_1,C_2} U = \ln(C_1) + \tfrac{1}{1+\rho}\ln(C_2) \text{ subject to } W \equiv X_1 + \tfrac{1}{1+r}X_2 \geq C_1 + \tfrac{1}{1+r}C_2$$

Lagrangian:

$$L = \ln(C_1) + \frac{1}{1+\rho}\ln(C_2) + \lambda\left[X_1 + \frac{1}{1+r}X_2 - C_1 - \frac{1}{1+r}C_2\right]$$

first-order conditions:

$$\frac{\partial L}{\partial C_1} = \frac{1}{C_1} - \lambda = 0$$

$$\frac{\partial L}{\partial C_2} = \frac{1}{1+\rho}\frac{1}{C_2} - \lambda\frac{1}{1+r} = 0$$

$$\frac{\partial L}{\partial \lambda} = X_1 + \tfrac{1}{1+r}X_2 - C_1 - \tfrac{1}{1+r}C_2 = 0$$

Using the first two gives:

$$C_1 = \frac{1+\rho}{1+r}C_2$$

substituting into third gives:

$$C_1 = \frac{1+\rho}{2+\rho}W \text{ and } C_2 = (1+r)\frac{1}{2+\rho}W$$

So direct consumer savings is given by:

$$B = X_1 - C_1 = \frac{1}{2+\rho}X_1 - \frac{1}{1+r}\frac{1+\rho}{2+\rho}X_2$$

note that $\frac{\partial B}{\partial r} > 0$ and $\frac{\partial B}{\partial X_2} < 0$

total savings is given by the sum of consumer savings (explicit savings) and firm savings (in the form of investment)

$$S_{tot} = B + I$$

Firm's Problem

$\underset{I}{\text{Max}}\, \Pi = Y_1 - K_2 + \tfrac{1}{1+r}[\alpha K_2^{\gamma} + (1-\delta)K_2]$ (assumes non-zero depreciation but also less than 100%)

first-order condition:

$$\frac{\partial \Pi}{\partial I} = -1 + \frac{1}{1+r}[\alpha\gamma K_2^{\gamma-1} + 1 - \delta] = 0$$

Solving for *I* gives:

$$K_2 = \left[\frac{\alpha\gamma}{\delta + r}\right]^{\frac{1}{1-\gamma}}$$

note that $\frac{\partial K_2}{\partial r} < 0$, $\frac{\partial K_2}{\partial Y_1} = 0$, $\frac{\partial K_2}{\partial K_1} = 0$ and $\frac{\partial K_2}{\partial \alpha} > 0$

Or in terms of investment, we note that $K_2 = (1-\delta)K_1 + I_1$, which gives:

$$I = \left[\frac{\alpha\gamma}{\delta + r}\right]^{\frac{1}{1-\gamma}} - (1-\delta)K_1$$

$\frac{\partial I}{\partial r} < 0$, $\frac{\partial I}{\partial Y_1} = 0$, $\frac{\partial I}{\partial K_1} < 0$ and $\frac{\partial I}{\partial \alpha} > 0$

We can view an increase in Y_1 as a temporary increase in output, while an increase in α is an expected increase in output in the future. A permanent increase in output would be the sum of both of these.

Adding-up Constraints

$$X_1 = Y_1 + (1-\delta)K_1 - K_2 = Y_1 - I$$

$$X_2 = \alpha K_2^{\gamma} + (1-\delta)K_2$$

Market Clearing
First, summarize our results:

$$CA_1 = \frac{1}{2+\rho} X_1 - \frac{1}{1+r}\frac{1+\rho}{2+\rho} X_2$$

With $X_1 \equiv Y_1 + (1-\delta)K_1 - K_2$; $X_2 \equiv \alpha K_2^{\gamma} + (1-\delta)K_2$; $K_2 = \left[\frac{\alpha\gamma}{\delta + r}\right]^{\frac{1}{1-\gamma}}$

There are three ways to have the market for savings and investment close: 1) a closed economy, 2) a small open economy, and 3) two large open economies.

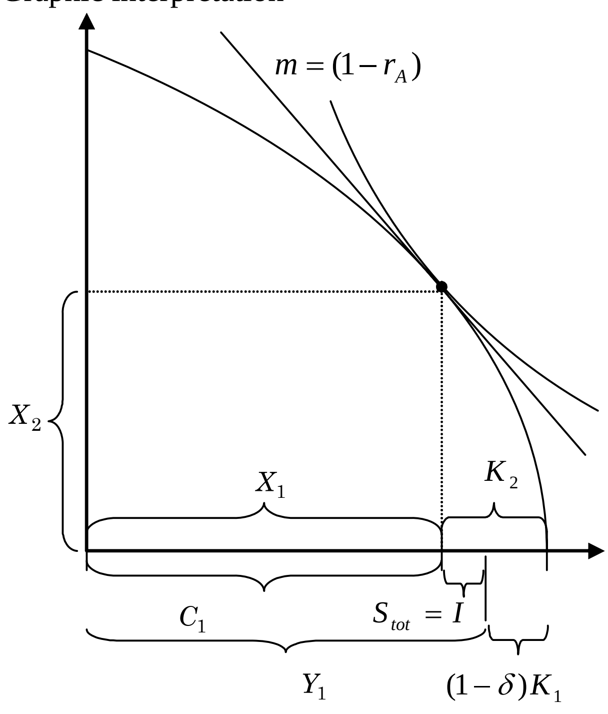
i) For a closed economy

$$CA_1 \equiv S_{tot} - I = B + I - I = B = 0$$

Solve for the value of r that satisfies this condition. This is the autarky interest rate, r_A.
This may need to be done numerically in many cases.

Consider how the autarky interest rate responds to changes in the exogenous variables (α, Y_1, K_1) and the parameters (δ, ρ, γ).

Graphic interpretation



ii) For a small open economy

Domestic interest rate is same as exogenous world interest rate: $r = r_w$

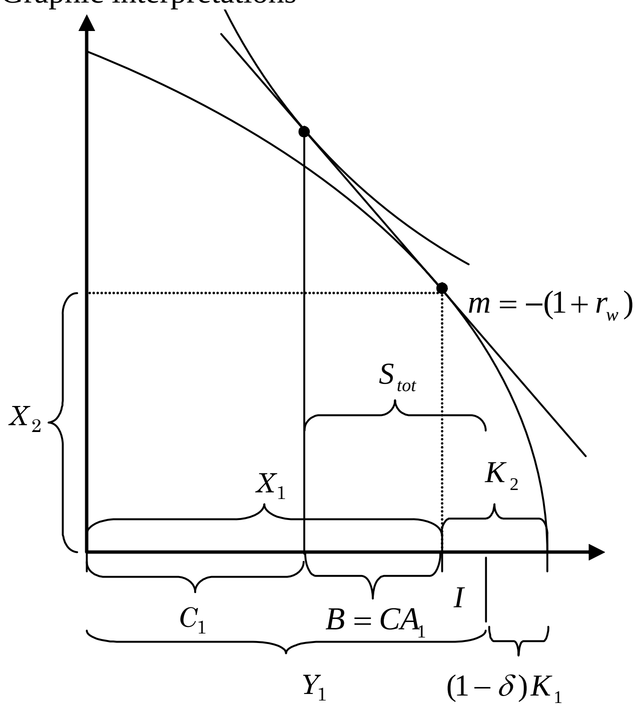
Solve for value of current account:

$$CA_1 = \frac{1}{2+\rho}\left[Y_1 + (1-\delta)K_1 - \left(\frac{\alpha\gamma}{\delta + r_w}\right)^{\frac{1}{1-\gamma}}\right] - \frac{1}{1+r_w}\frac{1+\rho}{2+\rho}\left[\alpha\left(\frac{\alpha\gamma}{\delta + r_w}\right)^{\frac{\gamma}{1-\gamma}} + (1-\delta)\left(\frac{\alpha\gamma}{\delta + r_w}\right)^{\frac{1}{1-\gamma}}\right]$$

Differentiating the above with respect to r_w, Y_1 and α gives:

$$\frac{dCA_1}{dr_w} > 0, \ \frac{dCA_1}{dY_1} > 0, \ \frac{dCA_1}{d\alpha} < 0, \ \frac{dCA_1}{dK_1} > 0$$

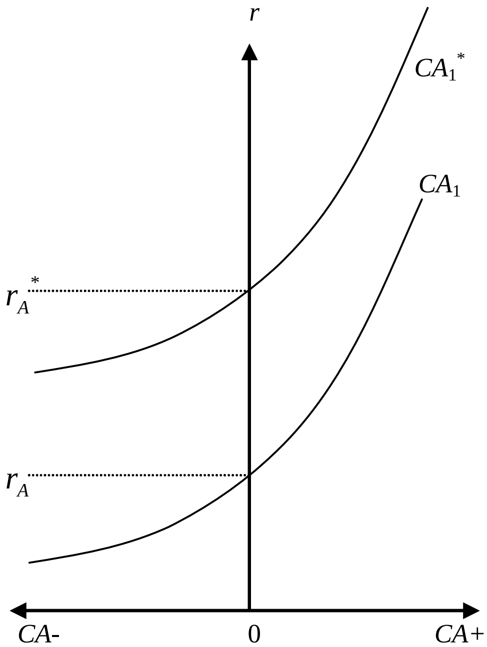
Graphic interpretations



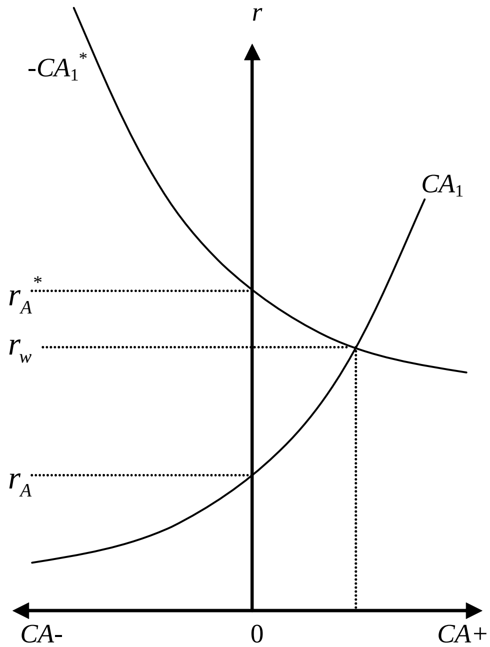
ii) For a large open economy
There are two large countries: the home country and the rest of the world (variable denoted with a *)

$CA_1 + CA_1^* = 0$
Solve for the value of r_w that satisfies this condition.

We can graph the home and current accounts as a function of the interest rate as shown below:



or also



Mathematically we solve this as follows:
$CA_1 + CA_1^* = 0$

$$\frac{1}{2+\rho}\left[Y_1 + (1-\delta)K_1 - \left(\frac{\alpha\gamma}{\delta + r_w}\right)^{\frac{1}{1-\gamma}}\right] - \frac{1}{1+r_w}\frac{1+\rho}{2+\rho}\left[\alpha\left(\frac{\alpha\gamma}{\delta + r_w}\right)^{\frac{\gamma}{1-\gamma}} + (1-\delta)\left(\frac{\alpha\gamma}{\delta + r_w}\right)^{\frac{1}{1-\gamma}}\right] +$$

$$\frac{1}{2+\rho^*}\left[Y_1^* + (1-\delta)K_1^* - \left(\frac{\alpha^*\gamma^*}{\delta^* + r_w}\right)^{\frac{1}{1-\gamma^*}}\right] - \frac{1}{1+r_w}\frac{1+\rho^*}{2+\rho^*}\left[\alpha^*\left(\frac{\alpha^*\gamma^*}{\delta^* + r_w}\right)^{\frac{\gamma^*}{1-\gamma^*}} + (1-\delta^*)\left(\frac{\alpha^*\gamma^*}{\delta^* + r_w}\right)^{\frac{1}{1-\gamma^*}}\right] = 0$$

Solve for r_w. Again this may require numerical techniques.

ADDING THE GOVERNMENT SECTOR (SMALL OPEN ECONOMY CASE)

Lump sum taxation gives the following gov't budget constraint:

$$T_1 + \frac{1}{1+r}T_2 \geq G_1 + \frac{1}{1+r}G_2$$

Consumers endowments are:

$$X_1 = Y_1 + (1-\delta)K_1 - K_2 - T_1$$

$$X_2 = \alpha K_2^{\gamma} + K_2 - T_2$$

The optimal level of capital is still given by $K_2 = \left[\frac{\alpha\gamma}{\delta + r}\right]^{\frac{1}{1-\gamma}}$

Total savings now is consumer savings plus firm savings less government borrowing. Hence the current account is now:

$$CA_1 = (B + I - BD) - I = B - (G_1 - T_1)$$

Holding government spending constant, consider alternative methods of finance:

1) Balanced budgets

$T_1 = G_1$; $T_2 = G_2$

$$CA_1 = \frac{1}{2+\rho}\left[Y_1 + (1-\delta)K_1 - K_2 - G_1\right] - \frac{1}{1+r_w}\frac{1+\rho}{2+\rho}\left[\alpha K_2^{\gamma} + (1-\delta)K_2 - G_2\right] - G_1 + G_1$$

2) Deficit in period 1

$T_1 = 0$; $T_2 = (1+r_w)G_1 + G_2$

$$CA_1 = \frac{1}{2+\rho}\left[Y_1 + (1-\delta)K_1 - K_2\right] - \frac{1}{1+r_w}\frac{1+\rho}{2+\rho}\left[\alpha K_2^{\gamma} + (1-\delta)K_2 - (1+r_w)G_1 - G_2\right] - G_1 + 0$$

3) Partial deficit in period 1:

$0 < T_1 < G_1$; $T_2 = (1+r_w)(G_1 - T_1) + G_2$

$$CA_1 = \frac{1}{2+\rho}\left[Y_1 + (1-\delta)K_1 - K_2 - T_1\right] - \frac{1}{1+r_w}\frac{1+\rho}{2+\rho}\left[\alpha K_2^{\gamma} + (1-\delta)K_2 - (1+r_w)(G_1 - T_1) - G_2\right] - G_1 + T_1$$

All the current account equations above can all be rewritten as:

$$CA_1 = \frac{1}{2+\rho}\left[Y_1 + (1-\delta)K_1 - \left(\frac{\alpha\gamma}{\delta + r_w}\right)^{\frac{1}{1-\gamma}}\right] - \frac{1}{1+r_w}\frac{1+\rho}{2+\rho}\left[\alpha\left(\frac{\alpha\gamma}{\delta + r_w}\right)^{\frac{\gamma}{1-\gamma}} + (1-\delta)\left(\frac{\alpha\gamma}{\delta + r_w}\right)^{\frac{1}{1-\gamma}}\right]$$

$$-\frac{1}{2+\rho}G_1 - \frac{1}{1+r_w}\frac{1+\rho}{2+\rho}G_2$$

This means the current account is the same regardless of how government spending is financed.

Also note that $\frac{\partial CA_1}{\partial G_1} < 0$ and $\frac{\partial CA_1}{\partial G_2} > 0$

Which implies that changes in the level of government spending in either period *will* change the current account.