Brigham Young University Department of Economics

Economics 459 - International Monetary Theory

Deriving the Equilibrium Approach to Exchange Rates

Start with our definition of the real exchange rate, $q \equiv eP^*/P$, and solve for the nominal rate: $e = qP/P^*$

Use domestic and foreign money market equilibria, $M/P = L(i,Y)$, solve for price levels and substitute to get:

$$e = q\frac{M}{M^*}\frac{L(i^*,Y^*)}{L(i,Y)}$$

Suppose the money demand function is log-linear, $\ln L = -ai + b\ln Y$, then we can rewrite this as:

$$e = q\frac{M}{M^*}\Lambda\left(i - i^*, \frac{Y^*}{Y}\right), \text{ where } \ln\Lambda = a(i - i^*) + b\ln(Y^*/Y); a,b \geq 0$$

Use the Fisher equations, $i = E\{r\} + E\{\pi\}$, to get:

$$e = q\frac{M}{M^*}\Lambda\left(E\{r\} + E\{\pi\} - (E\{r^*\} + E\{\pi^*\}), \frac{Y^*}{Y}\right)$$

Use real interest rate parity, $r - r^* = \delta + \rho$, to get:

$$e = q\frac{M}{M^*}\Lambda\left(E\{\delta\} + E\{\pi\} - E\{\pi^*\} + \rho, \frac{Y^*}{Y}\right), \text{ where } \delta \text{ is the percentage growth of } q.$$

Use the quantity theory of money to get $E\{\pi\} = E\{g_M\} - E\{g_Y\}$

$$e = q\frac{M}{M^*}\Lambda\left(E\{\delta\} + E\{g_M\} - E\{g_Y\} - (E\{g_M\} - E\{g_Y\}) + \rho, \frac{Y^*}{Y}\right)$$

Finally assume that the real exchange rate depends on relative levels of GDP, $q = q(Y^*/Y)$ and that we can approximate this function with a log-linear version, $\ln q = -c(\ln Y^* - \ln Y); c \geq 0$. This gives $\delta = -c(g_{Y^*} - g_Y)$.

Taking logs and substituting for ln q and δ gives:

$$\ln e = \ln q + \ln M - \ln M^* + \ln\Lambda$$

$$\ln e = -c(\ln Y^* - \ln Y) + \ln M - \ln M^* + a(i - i^*) + b(\ln Y^* - \ln Y)$$

$$\ln e = -c(\ln Y^* - \ln Y) + \ln M - \ln M^* + a(-c(g_{Y^*} - g_Y)$$
$$+ E\{g_M\} - E\{g_Y\} - E\{g_M\} + E\{g_Y\} + \rho) + b(\ln Y^* - \ln Y)$$

$$\ln e = (b - c)(\ln Y^* - \ln Y) + \ln M - \ln M^* + a[(1 - c)(E\{g_{Y^*}\} - E\{g_Y\}) + E\{g_M\} - E\{g_M\} + \rho]$$

$$\ln e = (b - c)\ln Y^* - (b - c)\ln Y + \ln M - \ln M^*$$
$$+ a(1 - c)E\{g_{Y^*}\} - a(1 - c)E\{g_Y\} + aE\{g_M\} - aE\{g_M\} + a\rho$$