Brigham Young University

Economics 459 – International Monetary Theory

Spring Term 2006

Final Exam key

1. Balance of Payments Accounting

Please indicate the effects of the following transactions on the Belugan balance of payments in the table below:

A Belugan National Petroleum purchases 50 million bugles worth of crude oil from Saudi Arabia and pays with a wire transfer from an account in a Swiss bank.

B Billionaire George Soros exchanges 300 million bugles in bank deposits at 1st Security Bank of Beluga for British Pound banknotes.

C The US sends a shipment of robotic explosive sniffing dogs valued at 20 million bugles to the Belugan government to help fight the war on terrorism.

D The Belugan Potato Farmers Cooperative barters a 40 million bugle shipment of potato harvesters for a shipload of Belugan Grey Spuds, a rare gourmet potato.

E Lacertilian Woolen Mills, the largest manufacturer of snakeskin socks in Beluga, sends a dividend check in the amount of 1 million bugles payable from 1st Security Bank of Beluga to Phil Church, a shareholders living in Florida.

	Current Account	Trade Account	Service Account	Net Factor Income	Unilateral Transfers	Capital Account	Net Increase Belugan holdings of Foreign Assets	Net Increase Foreign holdings of Belugan Assets
A	**-50 M**	**-50 M**	**0**	**0**	**0**	**+50 M**	**- (-50 M)**	**0**
B	**0**	**0**	**0**	**0**	**0**	**0**	**- (-300 M)**	**-300M**
C	**0**	**-20 M**	**0**	**0**	**+20 M**	**0**	**0**	**0**
D	**0**	**+40 M** **-40 M**	**0**	**0**	**0**	**0**	**0**	**0**
E	**0**	**0**	**0**	**-1 M**	**0**	**0**	**0**	**+1 M**

all quantities above are in millions of bugles

D could be interpreted as a purely domestic transaction, given the wording, and I gave full credit if you answered it that way

2. Long-run Exchange Rate Dynamics

Our primary trading partner is our neighboring country, Zalchistan. Both countries produce pretty much the same goods and we have both been growing at a robust 5% per year for the past decade or so. The Zalchistani central bank has just announced a series of monetary reforms aimed at keeping money supply growth rigidly fixed at 2% above the growth rate of real GDP. Most analysts think they are serious and the reforms are well-structured. We want the exchange rate between our two countries (Belugan Bugles per Zalchistani Zotney) to remain roughly fixed in the long run. What recommendations would you make to ensure this happens? Explain why your idea will work.

Since outputs in the two countries grow at the same rate, even if the goods were not perfect substitutes, the read exchange rate would be constant and δ^e=0. (*This insight was worth 4 pts.*)

In order for the exchange rate to remain constant, both Λ and M/M* need to remain constant. ɩ will remain constant as long as Y*/Y is constant and the growth rates of M, M*, Y & Y* don't change. So as long as M grows at the same rate as M* (7%) the exchange rate should remain constant in the long run.

This question did not require using the equilibrium approach to exchange rates, but using that model makes the analysis much easier to follow.

3. Real Exchange Rates

Our Bureau of Economic Statistics reports that per capita income in Beluga for the year 2006 was 78,400 bugles per person. We need to figure out how much this is worth in US dollars. The Penn World Tables report a PPP value for Beluga of 66.7 for 2000. The following data are available from various sources and may be of use:

	US CPI	Belugan CPI	exchange rate (bugles/dollar)
2000	107.9	115.2	11.44
2006	132.0	144.5	13.08

What is the value of per capita GDP in Beluga valued at US prices in 2006? Show your work in a step-by-step manner, explaining each step clearly.

1) We will treat Beluga as the home country, so the 2000 real exchange rate measured as Belugan baskets per US basket is 1/.667=1.5.

2) Find the real exchange rate in 2006. (*e* is the bugle per \$ nominal exchange rate, *p* is the Belugan price level, and *p is the US price level)**

$$\frac{q_{06}}{q_{00}} = \frac{e_{06}\,p^*_{06}}{p_{06}}\,\frac{p_{00}}{e_{00}\,p^*_{00}} \quad \textbf{or} \quad q_{06} = \frac{e_{06}}{e_{00}}\,\frac{p^*_{06}}{p^*_{00}}\,\frac{p_{00}}{p_{06}}\,q_{00} = \frac{13.08}{11.44}\,\frac{132.0}{107.9}\,\frac{115.2}{144.5}\,1.5 = 1.673$$

3) Convert 78,400 bugles to \$ by multiplying by the real rate and dividing by the nominal rate.

$$p^*_{06}Y = \frac{p_{06}q_{06}}{e_{06}}\,Y = \frac{q_{06}}{e_{06}}\,p_{06}Y = \frac{1.673}{13.08}\,78{,}400 = 10{,}028$$

So GDP per capita in Beluga was worth <u>\$10,028</u> in 2006.

As with the similar problem on the midterm, the biggest mistake here was getting the units mixed up. If we use the US as the home country, then q2000 is .667 and e2000 is 1/11.04, but if we use Beluga as the home country these are 1/.667 and 11.04, respectively. The most common mistake was using q2000=.667 and e2000=11.05.

4. Modeling the Current Account

Suppose that the utility function for Belugan households in period t is $\ln C_t$. Further suppose that household income is given by an endowment each period, X_t. Assuming households live forever, set up the typical household's maximization problem. Write the LaGrangian and its first-order conditions below.

For the sake of ease, roll principle and interest on B_0 into X_1.

$$L = \sum_{t=1}^{\infty} (\tfrac{1}{1+\rho})^{t-1} \ln C_t + \Lambda\left[\sum_{t=1}^{\infty} d_t X_t - \sum_{t=1}^{\infty} d_t C_t\right]; \quad d_t \equiv \begin{cases} 1 & \text{for } t=1 \\ \Pi_{s=2}^{t}(\tfrac{1}{1+r_s}) & \text{otherwise} \end{cases}$$

$$\frac{dL}{dC_t} = \left(\frac{1}{1+\rho}\right)^{t-1} C_t^{-1} - \Lambda d_t = 0$$

There is one of these conditions for each period from t=1 to infinity.

$$\frac{dL}{d\Lambda} = \sum_{t=1}^{\infty} d_t X_t - \sum_{t=1}^{\infty} d_t C_t = 0$$

There is one of the above

Then solve these conditions to get consumption each period as a function of utility parameters, the interest rate and the values of the endowments.

Solving the first foc for Λ gives:

$$\Lambda = \left(\frac{1}{1+\rho}\right)^{t-1} C_t^{-1} \frac{1}{d_t}; \ \forall t$$

Using consumption in period t and 1 gives:

$$C_1^{-\sigma} = \left(\frac{1}{1+\rho}\right)^{t-1} C_t^{-1} \frac{1}{d_t}; \ \forall t \ \ \textbf{or} \ \ C_t = [(1+\rho)^{t-1} d_t]^{=1} C_1$$

Substituting into the budget constraint gives:

$$W \equiv \sum_{t=1}^{\infty} d_t X_t = \sum_{t=1}^{\infty} [(1+\rho)^{t-1}]^{-1} C_1 \ \ \textbf{or} \ \ W = C_1 \sum_{t=1}^{\infty} (\tfrac{1}{1+\rho})^{t-1}$$

Using the sum of an infinite geometric sequence, $W = C_1 \dfrac{1}{1-(\frac{1}{1+\rho})}$**.**

and then solving for C_1 gives $C_1 = W \dfrac{\rho}{1+\rho}$

Which gives

$$\boxed{C_t = \frac{1}{(1+\rho)^{t-1} d_t} \frac{\rho}{1+\rho} \sum_{t=1}^{\infty} d_t X_t; \ d_t \equiv \begin{cases} 1 & \text{for } t=1 \\ \Pi_{s=2}^{t}(\tfrac{1}{1+r_s}) & \text{otherwise} \end{cases}}$$

Finally, show the current account as a function of the same inputs.

$$\boxed{CA_t = X_t - C_t = X_t - \frac{1}{(1+\rho)^t d_t} \frac{1}{\rho} \sum_{t=1}^{\infty} d_t X_t}$$

5. Numerical Analysis of the Current Account Model

Continuing from question 4, suppose that the rate of time preference (ρ) is 4% per year. Also suppose that the endowment stream starts with an initial value of $X_1 = 1000$ and grows at a constant rate of 3% each year.

What is the value of the current account in period 1 if Beluga is a small open economy facing a constant world interest rate ($\bar{r}$) of 5%? Show your work and how it relates to question 4 above.

If the interest rate is constant then the discount factor becomes $d_t = (\frac{1}{1+\bar{r}})^{t-1}$

Lifetime wealth is given by:

$$W \equiv \sum_{t=1}^{\infty} (\tfrac{1_t}{1+\bar{r}})^{t-1} \cdot X_1 \cdot (1+g)^{t-1} = \sum_{t=1}^{\infty} \left(\frac{1.03}{1.05}\right)^{t-1} 1000 = 1000\left(\frac{1}{1-\frac{1.03}{1.05}}\right) = 52{,}500$$

Using the formula for C_1 from problem 4:

$$C_1 = W\frac{1+\rho}{\rho} = 52500\frac{.04}{1.04} = 2019$$

So the current account is given by:

$$\boxed{CA_t = X_t - C_t = 1000 - 2019 = -1019}$$

What is the value of the interest rate between periods 1 & 2 (r_2) if Beluga is in autarky. Explain how you arrived at your answer. (Hint: Try looking at the Euler equation between periods t and t+1)

The Euler equation is derived from the t and t+1 foc's:

$$\Lambda = \left(\frac{1}{1+\rho}\right)^{t-1} C_t^{-1}\frac{1}{d_t} = \left(\frac{1}{1+\rho}\right)^{t} C_{t+1}^{-1}\frac{1}{d_{t+1}} \text{ or } C_t^{-1} = \left(\frac{1}{1+\rho}\right) C_{t+1}^{-1}(1+r_{t+1})$$

In autarky consumption equals the endowment and endowments are growing at a constant rate g=3%, so:

$$\frac{1}{X_1(1.03)^t} = \left(\frac{1}{1.04}\right)\frac{1}{X_1(1.03)^{t+1}}(1+r_{t+1})$$

Canceling the X_1 and $(1.03)^t$ gives:

$$1 = \left(\frac{1}{1.04}\right)\frac{1}{(1.03)}(1+r_{t+1}) \text{ or } 1+r_{t+1} = 1.04 \cdot 1.03 = 1.0712$$

**The interest rate is constant at 7.12% each period,
including between periods t & t+1**

This was poorly written. I took the s=.5 out when I wrote the answer key, because it was too hard for the final, but I forgot to take it out of the version I actually printed and handed out. No one got the right answer, even those who used s=1, as in problem 4 and those who derived everything from scratch with s=.5. So I just gave everyone 20 points for this question. Sorry for the hassle. But, hey, the teaching evaluations are already in!