

Brigham Young University Department of Economics  
**Economics 459 - International Monetary Theory**  
Gains from Trade in Financial Assets

**Identical Preferences & Differing Endowments**

General Analysis

Consider the following graph showing two individuals (H & F) with identical preferences and different endowments:

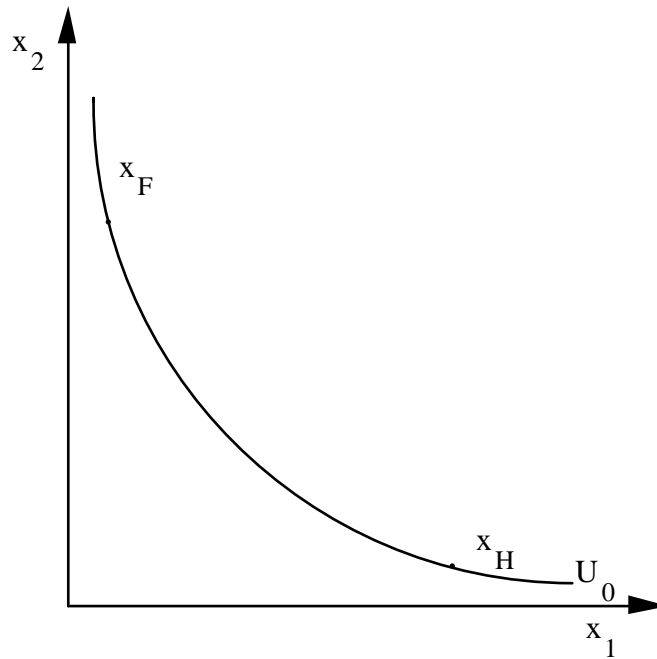


figure 1

As illustrated, H has more good 1 and F has more good 2. In the absence of trade each individual must consume his own endowment and gets level of utility  $U_0$ . Is there a way to make both individuals better off?

Suppose individual H gives some of his good 1 to individual F in exchange for some of individual F's good 2. We might get something like the following:

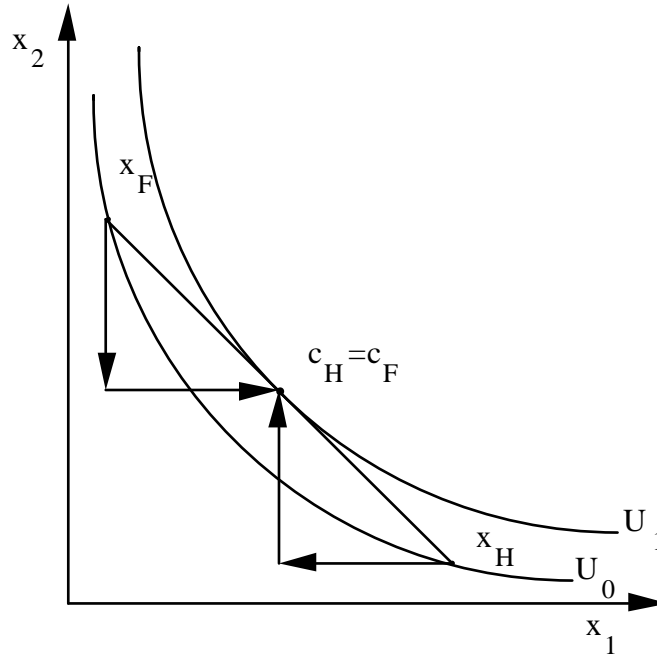


figure 2

As illustrated each individual is able to consume the same bundle of goods 1 & 2, and both individuals increase their utility level from  $U_0$  to  $U_1$ . (note: for balanced trade the two right triangles in the graph must be the same size and shape). The relative price of good 1 in terms of good 2 is given by the slope of the line connecting the two endowment points with the two consumption points.

In general there will be gains from trade as long as the slopes of each individual's indifference curves at the respective endowment points are not equal. Since the slope of the indifference curve for F is steeper at F's endowment point,  $x_F$ , than the slope of H's indifference curve at  $x_H$ ; at the margin F values good 1 more than H does.

### Trade over Time

We have assumed that goods 1 & 2 are two different goods. Suppose we now assume that good 1 is general consumption now and good two is general consumption in the future. As illustrated H is endowed with more goods now and F has more in the future. The common lifetime utility function might take the form:

$$U(c_1, c_2) = u(c_1) + \frac{1}{1+\rho} u(c_2) \quad (1)$$

where  $u(c)$  is a "momentary" utility function &  $\rho$  is a subjective discount rate (i.e. the rate at which individuals discount future utility).

The gains from trade will be illustrated exactly as before. H trades away some goods now for goods in the future; i.e. H saves. F trades future goods for goods now; i.e. F borrows. The slope of the trade triangles is  $1+R$  (where  $R$  is the interest rate) in this interpretation.

To realize the gains from trade in this context it is not enough to have trade in goods. In this case H has more of all goods than F does today. If H engages in balanced trade - swapping one good for another - H still has more goods. What H wants to do is reduce consumption today and increase it tomorrow. It can do this by buying a bond from F. If financial markets are closed then H & F cannot realize this type of a gain from trade, even if goods markets are free internationally.

### Trade under Uncertainty

Suppose there is uncertainty about the future, in particular, suppose that the world will end up in one of two states of nature. We could interpret goods 1 as goods in the first state of nature and good 2 as goods in the second state. Under this assumption, our illustration shows that H has a large endowment if state 1 occurs, and F has a large endowment if state two occurs. (note: any point along a  $45^\circ$  line from the origin would reflect a certain endowment in that the endowment is the same regardless of which state of nature occurs.)

The expected utility function that describes each individual's preferences might look like:

$$EU(c_1, c_2) = \pi u(c_1) + (1-\pi) u(c_2) \quad (2)$$

where  $u(c)$  is the utility function and  $\pi$  is the probability that state 1 occurs.

As before the gains from trade will be illustrated figure 2. H trades away some of his endowment in state 1 for goods in state 2. That is H & F engages in co-insurance. This type of a trade is accomplished by a contract between H & F that says, "if state 1 occurs H will pay F some goods, but if state 2 occurs F will pay goods to H."

Financial assets which pay only in one state of nature are known as Arrow-Debreu assets. If we adopt the practice of naming the asset after the state in which it pays off, then what we are illustrating in figure 2 is H trading some of the many type 1 A-D assets he has for some type 2 A-D assets.

Arrow-Debreu assets are not the only types of assets which support the gains from trade we have illustrated, however. Consider the notion of an equity. We will define an equity to be ownership of a particular asset bundle. Two bundles we can observe immediately from figure 1 are  $x_H$  and  $x_F$ , the endowment bundles for H & F. Figure 3 illustrates a trade in equities which gives the same net result as the trade in A-D assets illustrated in figure 2.

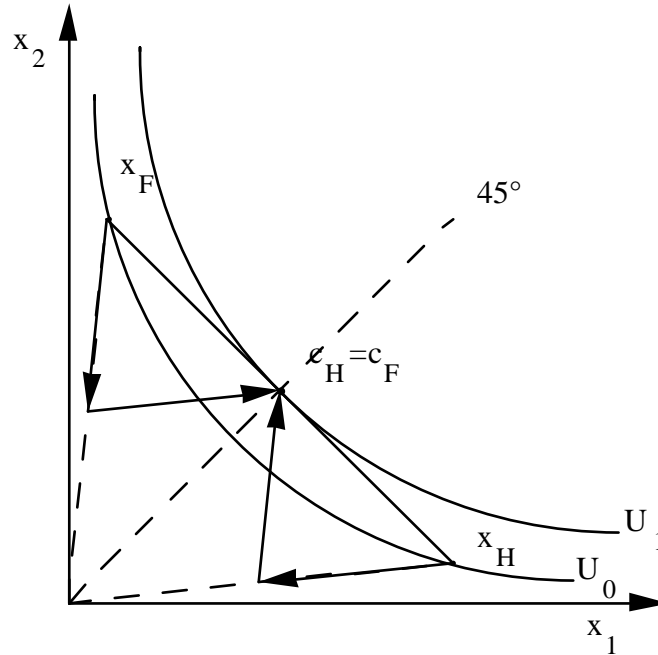


figure 3

If H were to give away some fraction of his ownership in  $x_H$  he would have a consumption point somewhere along a ray between the origin and  $x_H$ . If he gave exactly half his ownership away, for example, he would be at a point exactly half way between the origin and point  $x_H$ . If he were then to receive a gift of some fraction of F's ownership in  $x_F$ , his consumption point would move outward at a slope equal to that of the line between the origin and  $x_F$ .

As illustrated, H trades away exactly half his endowment for exactly half of F's. Both individuals consume the same amount and that amount is certain (i.e. lies on  $45^\circ$  line).

As before, to realize the gains from trade, it is not enough to have access to goods markets. Nor is access to bond markets enough, we need to have access to either equity or insurance markets as well.

### **Identical Endowments & Different Preferences**

#### General Analysis

Consider figure 4 which illustrates a case of identical endowments and differing preferences.

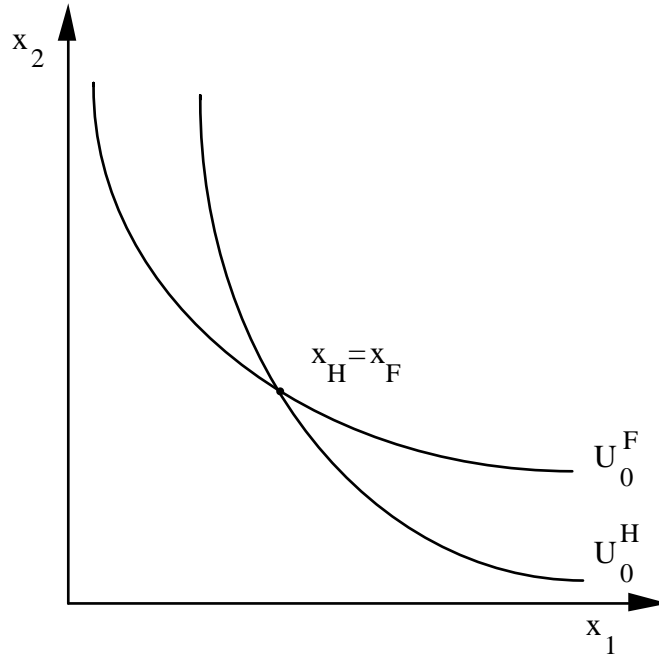


figure 4

As illustrated, F has a greater preference for good 2 than does H. This can be seen by noting that if a unit of good 2 is taken away from both individuals, F requires a greater reimbursement of good 1 than does H in order to remain indifferent.

Since the slopes of H & F's indifference curves differ, there are potential gains from trade. These gains can be illustrated by figure 5.

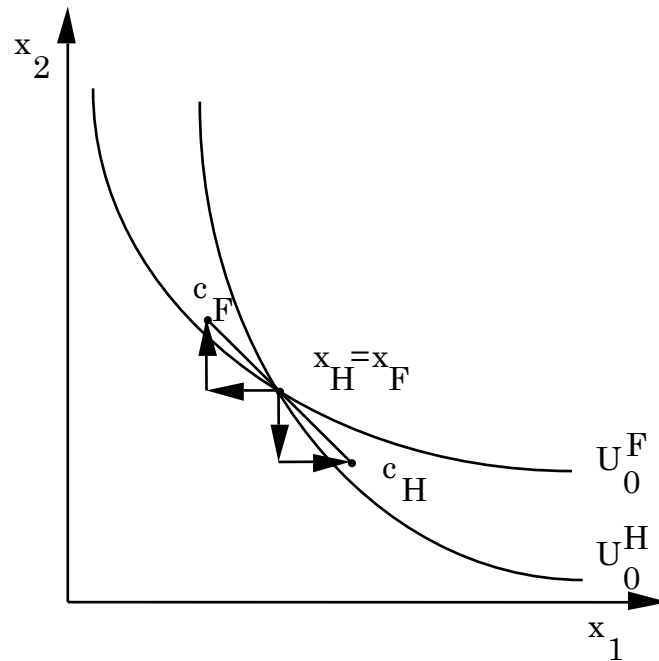


figure 5

Since F has a greater preference for good 2 and an identical endowment as H, F trades away some of his good 1 for some good 2. Both individuals have higher utility after this trade. As before the trade triangles must be of the same size & shape.

### Trade over Time

If good 1 is goods now and good 2 is future goods, we are considering a case where both countries have the same income profile. Preferences, however differ. Recall eq. (1) which expresses lifetime utility. Recall from 380 that the slope of an indifference curve is the marginal utility of good 1 divided by the marginal utility of good 2. Eq. (1) shows us that this value is:

$$\text{slope} = (1+\rho) \frac{u'(c_1)}{u'(c_2)} \quad (3)$$

Since the slope of F's indifference curve is flatter than H's, one way to interpret this difference in preferences is that  $\rho_F < \rho_H$ . Or that F's subjective discount rate is lower than H's, i.e. F is more patient. If F is more patient, then F gives up current consumption and saves while H borrows.

### Trade under Uncertainty

There are two ways to interpret figure 5 in the context of uncertainty

First, consider eq. (2) and assume that the utility functions,  $u(c)$ , are the same for both H & F. The slopes of the indifference curves could still be different. Note that the slope of the indifference curve under this interpretation is:

$$\text{slope} = \frac{\pi}{1-\pi} \frac{u'(c_1)}{u'(c_2)} \quad (4)$$

Hence, if  $\pi_F < \pi_H$  or F views state 1 as less likely to occur than H does, then F's indifference curve will be flatter than H's. F therefore trades away some of his state 1 endowment for some state 2 goods.

In this case if both individuals originally had endowments along the 45° line (i.e their endowments were certain), they would both consume risky consumption bundles after trade. Nonetheless, their expected utilities rise. The gains from trade come from the fact that they have different subjective assessments of the probabilities of each state.

Another interpretation of figure 5 is that the endowment point is below the 45° line, so that both individuals get more goods in state 1. If F is more risk averse his indifference curve will be flatter. To see this, consider two individuals with identical certain endowments. If we take away some state 2 goods and begin to make the endowment bundle more risky, we must compensate both individuals with greater amounts in state one to keep them indifferent. However the more risk averse individual will require greater amounts than the less risk averse one and will thus have an indifference curve that is flatter below the 45° line. Similar logic says that if will be steeper above the 45° line. This case is illustrated in figure 6.

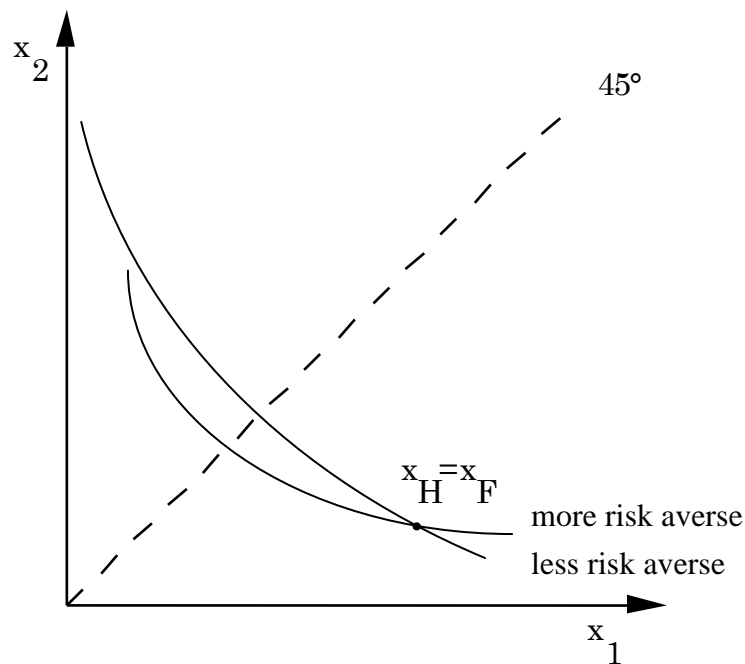


figure 6

Hence, figure 6 shows that if two individuals have the same risky endowment, but one is more risk averse than the other, then the risk averse individual will move trade away to a more certain consumption point, while the less risk averse individual trades away to a more risky consumption point.

Finally, recall the discussion earlier about trade in equities versus trade in Arrow-Debreu assets. Why is it that trade in equities is insufficient to realize the gains from trade illustrated in figures 4 – 6?