

Brigham Young University Department of Economics
Economics 458 - International Trade Theory
 Winter Semester 2002

Midterm Key

Due by Friday, March 1, 2002 at 5:00 p.m.

This test is open book and open notes. However, you may not consult with or discuss it with anyone while taking it. You need not take it in one sitting. Answer in the space provided. Try to keep your answers as precise and concise as possible, while still answering adequately. Be sure to answer in complete and understandable sentences and define any notation you introduce. Good luck!

- 1) Consider a world with two countries both in autarky.

Assume the following utility function for both countries:

$$U = X_C^{2/3} Y_C^{1/3}$$

Also assume the following PPF formula for both countries

$$Y_p = b - mX_p^2 \qquad Y_p = 200 - .02X_p^2$$

$$Y_p^* = b^* - m^* X_p^{*2} \qquad Y_p^* = 150 - .01X_p^{*2}$$

plot the PPFs on the next page

Derive excess demand functions for X in both countries and write their formulas below:

First, solve the firms' problems

$$\text{Max } P_X X_p + b - mX_p^2$$

$$\text{foc is: } P_X - 2bX_p = 0$$

solving for Xp and Yp gives:

$$X_p = \frac{P_X}{2b} = 25P_X \qquad Y_p = 200 - 12.5P_X^2$$

similarly solving for X*p & Y*p gives:

$$X_p^* = \frac{1}{2b^* P_X} = 50P_X \qquad Y_p^* = 150 - 25P_X^2$$

Next, solving the consumers' problems gives

$$X_C = \frac{2}{3}(X_p + Y_p / P_X) \qquad Y_C = \frac{1}{3}(P_X X_p + Y_p)$$

This gives the following excess demand functions:

$$E_X = .667(25P_X + 200 / P_X - 12.5P_X) - 25P_X = 133.33 / P_X - 16.67P_X$$

$$E_X^* = .667(50P_X + 150 / P_X - 25P_X) - 50P_X = 100 / P_X - 33.33P_X$$

Solve for the relative prices of X in autarky, using Y as the numeraire good for both countries.

$$\text{For home } E_X=0 \text{ implies } P_X^A = 2.828$$

$$\text{For foreign } E_X^*=0 \text{ implies } P_X^A = 1.732$$

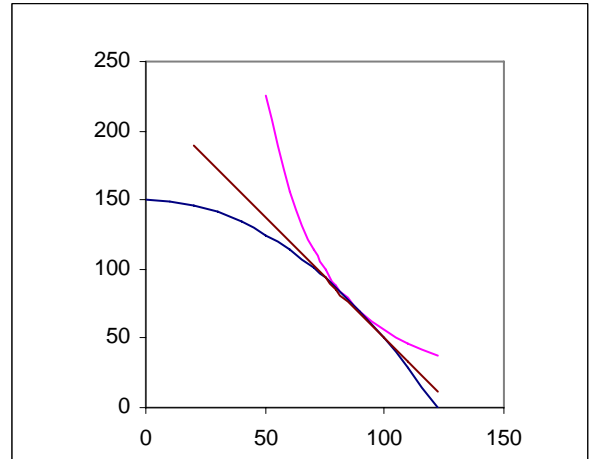
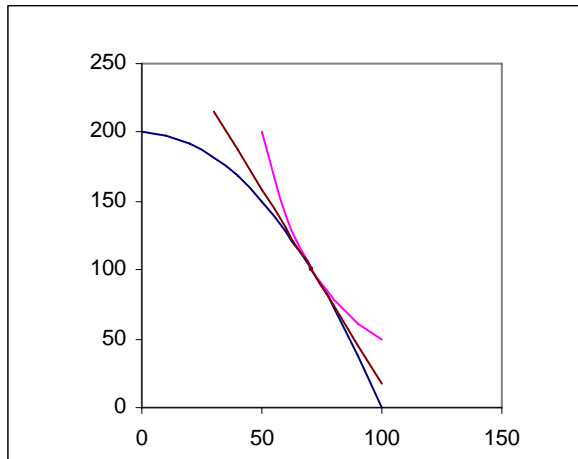
Report the productions of X & Y in autarky in both countries.

X=70.7 Y=100

X*=86.6 Y*=75

What are the autarky levels of utility?

U=79.36 U*=82.55



Illustrate the autarky equilibriums on the graphs above.

2) Now suppose that we open trade between these two countries.

Solve for the world relative price of X under free trade. Report it below.

$$E_X + E_X^* = 0$$

$$133.33/P_X - 16.67P_X + 100/P_X - 33.33P_X = 0$$

$$233.33 - 50P_X^2 = 0$$

positive root is: $P_X^T = 2.160$

Report the production of X & Y and the consumption of X & Y in both countries.

Production in H: X is 54.01 Y is 141.67

Consumption in H: X is 79.76 Y is 86.03

Production in F: X is 108.01 Y is 33.33

Consumption in F: X is 82.34 Y is 88.80

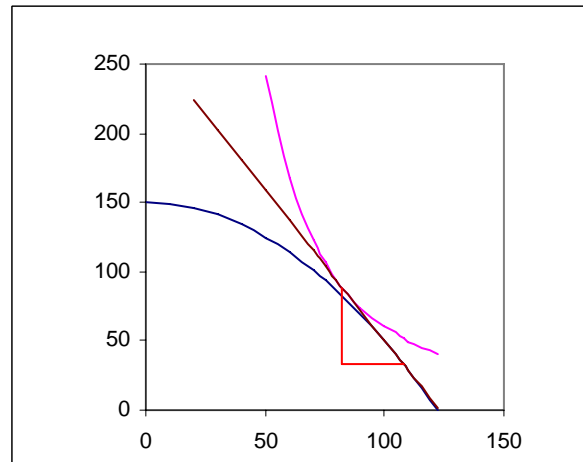
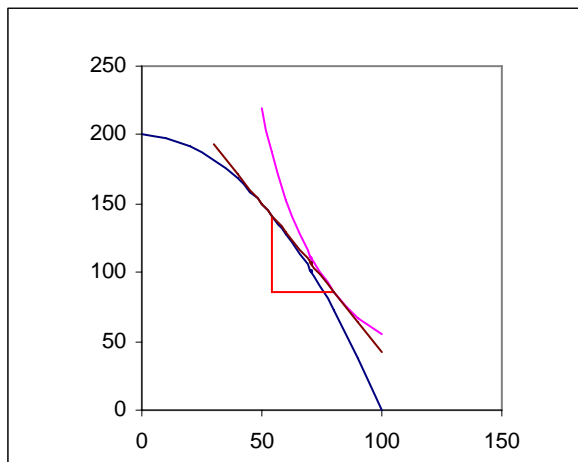
Report imports and exports of X & Y.

25.76 units of X flow from F to H

55.47 units of Y flow from H to F

Report the levels of utility in both countries.

U = 81.8 U*=84.43



Illustrate the free trade equilibrium on the graphs above.

3) Next, suppose that the home country institutes a 25% subsidy on the production of good X

Let the domestic net prices that consumers perceive be denoted by q 's and the prices that firms perceive be denoted with p 's. Write down relations between the relative prices faced by both producers and consumers in both countries. (For example, there is no subsidy in the foreign country so we can write: $p^* = q^*$)

To keep things simple, ignore cost to the home government of financing the subsidy.

Solve for the world relative price of X under free trade with this subsidy. Report it below.

We have following relation between prices:

$$q_X = P_X^* = q_X^* \equiv q \quad \& \quad P_X = 1.25q$$

Home country's firms problem looks like

$$\text{Max } q(1.25)X_p + 200 - .02X_p^2$$

for is: $1.25q = .04X_p$ **which gives:** $X_p = 31.25q$ **and** $Y_p = 200 - 19.53q^2$

Home Consumer's problem yields the same shares as before, substituting the above gives:

$$X_C = \frac{2}{3}(31.25q + 200/q - 19.53q)$$

and this yields the following excess demand curve:

$$E_X = \frac{2}{3}(11.72q + 200/q) - 31.25q = 133.33/q - 23.44q$$

the foreign excess demand is the same as before:

$$E_X^* = 100/P_X - 33.33P_X$$

$$E_X + E_X^* = 0 \quad \text{gives:}$$

$$133.33/q - 23.44q + 100/P_X - 33.33P_X = 0$$

which can be solved for q to give:

$$q = 2.0273 \quad \text{and the home producer price is thus, } P_X = 2.5342$$

Report the production of X & Y and the consumption of X & Y in both countries.

Production in H: X is 63.34 Y is 119.76

Consumption in H: X is 81.66 Y is 82.64

Production in F: X is 101.35 Y is 47.28

Consumption in F: X is 83.16 Y is 84.16

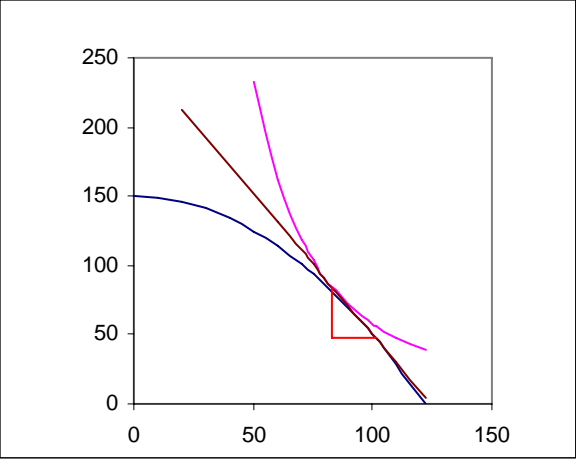
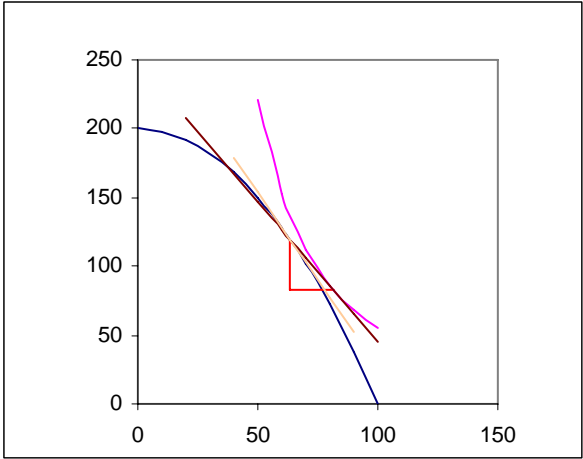
Report imports and exports of X & Y.

18.31 units of X flow from F to H

37.12 units of Y flow from H to F

Report the levels of utility in both countries.

$$U = 81.94 \quad U^* = 83.49$$



Illustrate the free trade / subsidy equilibrium on the graphs above.

- 4) Summarize below what you learned from the above exercise. What does the model illustrate? How critically does what you learned depend upon the particulars of this model?