TECHNICAL APPENDIX

T-1 Derivation of Monopolistic Competition Demand and Price Equations in Two-Country OLG Model

In this section, I derive the individual differentiated-good demand equations for the two-country overlapping generations model described in this paper as well expressions for aggregate demand and aggregate prices. This follows the intuition of the closed economy derivation as first proposed by Dixit and Stiglitz (1977). The difference here is that aggregate consumption for each individual in time $t+1$ is a Home-biased composite of both Home and Foreign goods. I assume that consumers only care about Home-biased aggregate consumption C_{t+1} and that the elasticity of substitution among Home differentiated goods and the elasticity of substitution among Foreign differentiated goods is a constant ε that is symmetric across countries. However, I assume that the elasticity of substitution between a unit Home aggregate consumption and a unit of Foreign aggregate consumption is, in general, not equal to the elasticity of substitution among individually differentiated goods.

Let ρ be the constant elasticity of substitution between a unit of aggregate Home-country consumption and a unit of aggregate Foreign-country consumption. And let ε be the constant elasticity of substitution among the differentiated goods of each country. A realistic assumption is that the elasticity among the goods of a specific country is greater than the elasticity between a units of aggregate consumption from each country $\varepsilon > \rho$.

The form of the CES consumption aggregator with a Home-bias term θ is represented by the following two equations for the Home country and the Foreign country, respectively.

$$C_{t+1} \equiv \left[(1-\theta)^{\frac{1}{\rho}}\left(C^h_{t+1}\right)^{\frac{\rho-1}{\rho}} + \theta^{\frac{1}{\rho}}\left(C^f_{t+1}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{T.1.1}$$

$$C^*_{t+1} \equiv \left[(1-\theta)^{\frac{1}{\rho}}\left(C^{f*}_{t+1}\right)^{\frac{\rho-1}{\rho}} + \theta^{\frac{1}{\rho}}\left(C^{h*}_{t+1}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{T.1.2}$$

where $\rho \geq 0$ is the elasticity of substitution between Home and Foreign aggregate consumption, $\theta \in \left[0, \frac{1}{2}\right]$ parameterizes the degree of Home-bias in both countries, and C^h_{t+1}, C^f_{t+1}, C^{h*}_{t+1}, and C^{f*}_{t+1} represent aggregate consumption of Home produced and Foreign produced goods by Home and Foreign consumers, respectively. The exponent on the Home-bias parameter $1/\rho$ is merely an *ad hoc* functional form that makes the solutions more clean. An alternative would be an exponent of 1. From this point on, I only provide the derivation for the Home country, but the derivation for the Foreign country is completely symmetric.

If Home consumer purchases individual differentiated goods consumption $c_{t+1}(z)$ from Home producer z and $c_{t+1}(z^*)$ from Foreign producer z^*, aggregate consumption of goods from each country C^h_{t+1} and C^f_{t+1} can be defined by the Dixit-Stiglitz CES aggregator:

$$\text{(T.1.3)} \qquad C^h_{t+1} \equiv \left(\int_0^1 c_{t+1}(z)^{\frac{\varepsilon-1}{\varepsilon}} \, dz \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall t$$

$$\text{(T.1.4)} \qquad C^f_{t+1} \equiv \left(\int_0^1 c_{t+1}(z^*)^{\frac{\varepsilon-1}{\varepsilon}} \, dz^* \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall t$$

where $\varepsilon \geq 0$ represents the elasticity of substitution among all differentiated goods of a given country.[14] The individual demand equations for each differentiated good $c_{t+1}(z)$ and $c_{t+1}(z^*)$ for all z and z^* result from minimizing the cost of consuming given aggregate levels of consumption C^h_{t+1} and C^f_{t+1} by choosing the optimal consumption bundle $c_{t+1}(z)$ and $c_{t+1}(z^*)$ given individual prices $p_{t+1}(z)$ and $p_{t+1}(z^*)$.[15]

$$\text{(T.1.5)} \qquad \begin{aligned} &\min_{c_{t+1}(z), c_{t+1}(z^*)} \int_0^1 p_{t+1}(z) c_{t+1}(z) \, dz + e_t \int_0^1 p_{t+1}(z^*) c_{t+1}(z^*) \, dz^* ... \\ &\text{subject to} \quad C^h_{t+1} \leq \left[\int_0^1 c_{t+1}(z)^{\frac{\varepsilon-1}{\varepsilon}} \, dz \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &\qquad \text{and} \quad C^f_{t+1} \leq \left[\int_0^1 c_{t+1}(z^*)^{\frac{\varepsilon-1}{\varepsilon}} \, dz^* \right]^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned}$$

where the exchange rate e_t is lagged one period due to the portfolio decision being made in the period previous to consumption. The Lagrangian is the following:

$$\text{(T.1.6)} \quad \begin{aligned} \mathcal{L} = &\int_0^1 p_{t+1}(z) c_{t+1}(z) \, dz + e_t \int_0^1 p_{t+1}(z^*) c_{t+1}(z^*) \, dz^* ... \\ &+ \lambda_h \left(C^h_{t+1} - \left[\int_0^1 c_{t+1}(z)^{\frac{\varepsilon-1}{\varepsilon}} \, dz \right]^{\frac{\varepsilon}{\varepsilon-1}} \right) ... \\ &+ \lambda_f \left(C^f_{t+1} - \left[\int_0^1 c_{t+1}(z^*)^{\frac{\varepsilon-1}{\varepsilon}} \, dz^* \right]^{\frac{\varepsilon}{\varepsilon-1}} \right) \end{aligned}$$

Because the Lagrange multipliers λ_h and λ_f have the interpretation of being the marginal cost of an extra unit of aggregated country-specific consumption in terms of Home-country currency, λ_k is the price of aggregated country-specific consumption P^k_{t+1} for $k = h, f$. That is, P^h_t is the Home country price index of Home produced

[14]Appendix T-3 details some of the various forms that this CES aggregator can take resulting from different specifications of ε.

[15]The dual problem of maximizing the level of aggregate consumption subject to a budget constraint of expenditures being less than the currency held at the time of exchange does not yield the same result because the multiplier on the budget constraint does not have the interpretation as the price of an extra unit of aggregate consumption.

goods consumed at Home, and P_t^f is the import price index. The Lagrangian is now given by:

$$\text{(T.1.7)}\quad \mathcal{L} = \int_0^1 p_{t+1}(z)c_{t+1}(z)\,dz + e_t \int_0^1 p_{t+1}(z^*)c_{t+1}(z^*)\,dz^* ... + P_{t+1}^h \left(C_{t+1}^h - \left[\int_0^1 c_{t+1}(z)^{\frac{\varepsilon-1}{\varepsilon}}\,dz \right]^{\frac{\varepsilon}{\varepsilon-1}} \right) ... + e_t P_{t+1}^f \left(C_{t+1}^f - \left[\int_0^1 c_{t+1}(z^*)^{\frac{\varepsilon-1}{\varepsilon}}\,dz^* \right]^{\frac{\varepsilon}{\varepsilon-1}} \right)$$

Because the constraints always bind, the first order conditions are:

$$\text{(T.1.8)}\qquad p_{t+1}(z) = P_{t+1}^h \left(\int_0^1 c_{t+1}(z)^{\frac{\varepsilon-1}{\varepsilon}}\,dz \right)^{\frac{1}{\varepsilon-1}} c_{t+1}(z)^{-\frac{1}{\varepsilon}} \quad \forall\, t, z$$

$$\text{(T.1.9)}\qquad p_{t+1}(z^*) = P_{t+1}^f \left(\int_0^1 c_{t+1}(z^*)^{\frac{\varepsilon-1}{\varepsilon}}\,dz^* \right)^{\frac{1}{\varepsilon-1}} c_{t+1}(z^*)^{-\frac{1}{\varepsilon}} \quad \forall\, t, z^*$$

$$\text{(T.1.10)}\qquad C_{t+1}^h = \left(\int_0^1 c_{t+1}(z)^{\frac{\varepsilon-1}{\varepsilon}}\,dz \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall t$$

$$\text{(T.1.11)}\qquad C_{t+1}^f = \left(\int_0^1 c_{t+1}(z^*)^{\frac{\varepsilon-1}{\varepsilon}}\,dz^* \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall t$$

Solving for $c_{t+1}(z)$ and $c_{t+1}(z^*)$ in (T.1.18) and (T.1.19) and plugging in the constraints from (T.1.10) and (T.1.11) which are simply the definitions of aggregated country-specific consumption from (T.1.3) and (T.1.4), the demand for each country's individual differentiated goods take the following form:

$$\text{(T.1.12)}\qquad c_{t+1}(z) = \left(\frac{p_{t+1}(z)}{P_{t+1}^h} \right)^{-\varepsilon} C_{t+1}^h \quad \forall t, z$$

$$\text{(T.1.13)}\qquad c_{t+1}(z^*) = \left(\frac{p_{t+1}(z^*)}{P_{t+1}^f} \right)^{-\varepsilon} C_{t+1}^f \quad \forall t, z^*$$

Plugging (T.1.12) and (T.1.13) back into (T.1.10) and (T.1.11) and solving for P_{t+1}^h and P_{t+1}^f, respectively, gives the analogous expression for the price of aggregated country-specific consumption.

$$\text{(T.1.14)}\qquad P_{t+1}^h = \left(\int_0^1 p_{t+1}(z)^{1-\varepsilon}\,dz \right)^{\frac{1}{1-\varepsilon}} \quad \forall t$$

(T.1.15) $$P_{t+1}^{f} = \left(\int_0^1 p_{t+1}(z^*)^{1-\varepsilon}\, dz^* \right)^{\frac{1}{1-\varepsilon}} \qquad \forall t$$

As in the cost minimization problem in (T.1.5), the Home consumer seeks to minimize total expenditure subject to a given level of aggregate consumption.

(T.1.16)

$$\min_{C_{t+1}^h, C_{t+1}^f} \; P_{t+1}^h C_{t+1}^h + e_t P_{t+1}^f C_{t+1}^f \quad \text{s.t.} \quad C_{t+1} \leq \left[(1-\theta)^{\frac{1}{\rho}} \left(C_{t+1}^h\right)^{\frac{\rho-1}{\rho}} + \theta^{\frac{1}{\rho}} \left(C_{t+1}^f\right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$$

The Lagrangian for this problem is:

(T.1.17)

$$\mathcal{L} = P_{t+1}^h C_{t+1}^h + e_t P_{t+1}^f C_{t+1}^f + P_{t+1} \left(C_{t+1} - \left[(1-\theta)^{\frac{1}{\rho}} \left(C_{t+1}^h\right)^{\frac{\rho-1}{\rho}} + \theta^{\frac{1}{\rho}} \left(C_{t+1}^f\right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \right)$$

where P_{t+1} is the multiplier on the constraint and represents the marginal cost of an extra unit of aggregate consumption. So P_{t+1} is interpreted as the price of aggregate consumption. The first order conditions are the following:

(T.1.18) $$P_{t+1}^h = P_{t+1} \left[\frac{(1-\theta) C_{t+1}}{C_{t+1}^h} \right]^{\frac{1}{\rho}}$$

(T.1.19) $$e_t P_{t+1}^f = P_{t+1} \left[\frac{\theta C_{t+1}}{C_{t+1}^f} \right]^{\frac{1}{\rho}}$$

(T.1.20) $$C_{i,t+1} = \left[(1-\theta)^{\frac{1}{\rho}} \left(C_{t+1}^h\right)^{\frac{\rho-1}{\rho}} + \theta^{\frac{1}{\rho}} \left(C_{t+1}^f\right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$$

Dividing (T.1.18) by (T.1.19) gives the following relationship:

(T.1.21) $$\frac{P_{t+1}^h}{e_t P_{t+1}^f} \left(\frac{C_{t+1}^h}{C_{t+1}^f} \right)^{\frac{1}{\rho}} = \left(\frac{1-\theta}{\theta} \right)^{\frac{1}{\rho}}$$

Notice that in the Cobb-Douglas or log utility case when $\rho = 1$, the ratio of Home consumption expenditure to Foreign consumption expenditure is a constant.[16] Also, note that solving (T.1.18) and (T.1.19) for C_{t+1}^h and C_{t+1}^f, respectively, gives Home demand equations for aggregate consumption of Home goods and aggregate consumption of Foreign goods.

(T.1.22) $$C_{t+1}^h = (1-\theta) \left(\frac{P_{t+1}^h}{P_{t+1}} \right)^{-\rho} C_{t+1}$$

[16] The outcome when $\rho = 1$ is different from the case when $\rho \in (0, \infty)$ and $\rho \neq 1$ in a critical way. As is shown in Appendix T-4, in the case when $\rho \neq 1$, an international Bertrand duopoly situation develops between monetary authorities, and the world equilibrium is (x, x^*). [*This last sentence may not be correct.*]

$$C^f_{t+1} = \theta \left(\frac{e_t P^f_{t+1}}{P_{t+1}} \right)^{-\rho} C_{t+1} \tag{T.1.23}$$

These demand equations are analogous to the individual demand equations in (T.1.12) and (T.1.13), except that they include the Home-bias parameter.

The expression for the aggregate price index P_{t+1} of the Home consumption over aggregate Home and Foreign consumption is found by rewriting (T.1.20) as:

$$C_{t+1} = \left[\left(\frac{1-\theta}{C^h_{t+1}} \right)^{\frac{1}{\rho}} C^h_{t+1} + \left(\frac{\theta}{C^f_{t+1}} \right)^{\frac{1}{\rho}} C^f_{t+1} \right]^{\frac{\rho}{\rho-1}} \tag{T.1.24}$$

Then, substituting the expressions for $\left([1-\theta]/C^h_{t+1}\right)^{1/\rho}$ and $\left(\theta/C^f_{t+1}\right)^{1/\rho}$ from (T.1.18) and (T.1.19) into (T.1.24) gives the expression for aggregate expenditures which is implied by the cost minimization problem in (T.1.16):

$$P_{t+1} C_{t+1} = P^h_{t+1} C^h_{t+1} + e_t P^f_{t+1} C^f_{t+1} \tag{T.1.25}$$

Now divide (T.1.25) by aggregate consumption C_{t+1} and plug in the expressions for C^h_{t+1}/C_{t+1} and C^f_{t+1}/C_{t+1} from (T.1.18) and (T.1.19). The resulting expression for aggregate price is:

$$P_{t+1} = \left[(1-\theta) \left(P^h_{t+1} \right)^{1-\rho} + \theta \left(e_t P^f_{t+1} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}} \tag{T.1.26}$$

Note that this expression is the Home country CPI and is analogous to the within-country price aggregator in equation (T.1.14) but with the inclusion of the Home-bias parameter θ.

In the case of Cobb-Douglas aggregation over aggregate Home consumption and aggregate Foreign consumption ($\rho = 1$), the expression for aggregate price is:

$$P_{t+1} = \frac{1}{(1-\theta)^{1-\theta} \theta^\theta} \left(P^h_{t+1} \right)^{1-\theta} \left(e_t P^f_{t+1} \right)^\theta \tag{T.1.27}$$

and total aggregate expenditure is given by:

$$P_{t+1} C_{t+1} = \frac{1}{(1-\theta)^{1-\theta} \theta^\theta} \left(P^h_{t+1} C^h_{t+1} \right)^{1-\theta} \left(e_t P^f_{t+1} C^f_{t+1} \right)^\theta \tag{T.1.28}$$

T-2 Derivations

Derivation 1 (Demand for differentiated good z). The demand function for individual differentiated good z from (14) is derived in the following way:

$$
\begin{aligned}
d_t(z) \equiv c_t(z) + c_t^*(z) = \left(\frac{p_t(z)}{P_t^h}\right)^{-\varepsilon} C_t^h + \left(\frac{p_t(z)}{P_t^h}\right)^{-\varepsilon} C_t^{h*} \\
= \left(\frac{p_t(z)}{P_t^h}\right)^{-\varepsilon} \frac{m_t^h}{P_t^h} + \left(\frac{p_t(z)}{P_t^h}\right)^{-\varepsilon} \frac{m_t^{h*}}{P_t^h} \\
= \left(\frac{p_{i,t}(z)}{P_t^h}\right)^{-\varepsilon} \frac{m_t^h + m_t^{h*}}{P_t^h} = \left(\frac{p_{i,t}(z)}{P_t^h}\right)^{-\varepsilon} \frac{M_t}{P_t^h} \\
d_t(z) = \left(\frac{p_{i,t}(z)}{P_t^h}\right)^{-\varepsilon} \frac{xM_{t-1}}{P_t^h}
\end{aligned}
$$

Derivation 2 (Steady State Equilibrium Home and Foreign CPI growth rates). The steady state equilibrium Home and Foreign country CPI growth rates as shown in (**??**) and (**??**) are derived in the following way. The Home CPI level is derived in Appendix T-1, and takes the following form as in (10):

$$
P_{t+1} = \frac{1}{(1-\theta_h)^{1-\theta_h}\theta_h^{\theta_h}} \left(P_{t+1}^h\right)^{1-\theta_h} \left(e_t P_{t+1}^f\right)^{\theta_h}
$$

Dividing P_{t+1} by P_t gives the following expression for the Home country CPI growth rate:

$$
\begin{aligned}
\frac{P_{t+1}}{P_t} &= \left(\frac{P_{t+1}^h}{P_t^h}\right)^{1-\theta_h} \left(\frac{e_t P_{t+1}^f}{e_{t-1} P_t^f}\right)^{\theta_h} \\
&= (x)^{1-\theta_h} \left(\frac{e_t}{e_{t-1}} x^*\right)^{\theta_h}
\end{aligned}
$$

Using the currency exchange market clearing condition (26) and plugging in the equilibrium expressions for m_t^f and m_t^{h*}, the steady state equilibrium expression for the growth rate of the exchange rate is:

$$
\frac{e_t}{e_{t-1}} = \frac{x}{x^*}
$$

Thus, the expression for the steady state equilibrium CPI growth rate in the Home country is:

$$
\frac{P_{t+1}}{P_t} = x
$$

And by symmetry, the steady state equilibrium CPI growth rate in the Foreign country is:

$$
\frac{P_{t+1}^*}{P_t^*} = x^*
$$

It is the steady-state exchange rate growth expression that cancels out the effects of the other country's prices in each CPI growth rate expression.

Derivation 3 (**Sign of parameter objects and derivatives with respect to θ_h and θ_f**)**.** Here I derive the derivatives of the parameter summary objects with respect to θ_h and θ_f. A review of the objects and their representation is the following:

$$\begin{aligned}
\Delta_h &= (1-\theta_h)(1-\sigma)-\xi \\
\Delta_f &= (1-\theta_f)(1-\sigma)-\xi \\
\Sigma_h &= \theta_h(1-\sigma) \\
\Sigma_f &= \theta_f(1-\sigma) \\
\Omega_h &= \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\chi\xi}{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}(\theta_f)^{\theta_h(1-\sigma)}} \\
\Omega_f &= \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\chi\xi}{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}(\theta_h)^{\theta_f(1-\sigma)}} \\
\Omega_H &= (\Omega_h)^{\frac{\Delta_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}\,(\Omega_f)^{\frac{-\Sigma_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \\
\Omega_F &= (\Omega_f)^{\frac{\Delta_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}\,(\Omega_h)^{\frac{-\Sigma_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}
\end{aligned}$$

The signs of the representative parameter objects and their derivatives with respect to θ_h and θ_f are the following:

$$\Delta_h < 0 \text{ always}, \quad \frac{\partial\Delta_h}{\partial\theta_h} = -(1-\sigma) > 0 \text{ when } \sigma > 1$$

$$\Delta_f < 0 \text{ always}, \quad \frac{\partial\Delta_f}{\partial\theta_f} = -(1-\sigma) > 0 \text{ when } \sigma > 1$$

$$\Sigma_h < 0 \text{ when } \sigma > 1 \text{ and } \theta_h > 0, \quad \frac{\partial\Sigma_h}{\partial\theta_h} = 1-\sigma < 0 \text{ when } \sigma > 1$$

$$\Sigma_f < 0 \text{ when } \sigma > 1 \text{ and } \theta_f > 0, \quad \frac{\partial\Sigma_f}{\partial\theta_f} = 1-\sigma < 0 \text{ when } \sigma > 1$$

$\Omega_h > 0$ when $\theta_f > 0$,

$$\frac{\partial\Omega_h}{\partial\theta_h} = \Omega_h\frac{\partial log(\Omega_h)}{\partial\theta_h} = \Omega_h(1-\sigma)\left[1+log\left(\frac{1-\theta_h}{\theta_f}\right)\right] < 0 \text{ when } \sigma > 1 \text{ and } \theta_f > 0$$

$$\frac{\partial\Omega_h}{\partial\theta_f} = \Omega_h\frac{\partial log(\Omega_h)}{\partial\theta_f} = -\Omega_h(1-\sigma)\frac{\theta_h}{\theta_f} > 0 \text{ when } \sigma > 1 \text{ and } \theta_h > 0$$

$\Omega_f > 0$ when $\theta_h > 0$,

$$\frac{\partial\Omega_f}{\partial\theta_f} = \Omega_f\frac{\partial log(\Omega_f)}{\partial\theta_f} = \Omega_f(1-\sigma)\left[1+log\left(\frac{1-\theta_f}{\theta_h}\right)\right] < 0 \text{ when } \sigma > 1 \text{ and } \theta_h > 0$$

$$\frac{\partial\Omega_f}{\partial\theta_h} = \Omega_f\frac{\partial log(\Omega_f)}{\partial\theta_h} = -\Omega_f(1-\sigma)\frac{\theta_f}{\theta_h} > 0 \text{ when } \sigma > 1 \text{ and } \theta_f > 0$$

$$\Delta_h\Delta_f - \Sigma_h\Sigma_f = (1-\theta_h-\theta_f)(1-\sigma)^2 - (1-\sigma)\xi(2-\theta_h-\theta_f) + \xi^2 > 0 \text{ always}$$

$$\frac{\partial(\Delta_h\Delta_f-\Sigma_h\Sigma_f)}{\partial\theta_h} = \frac{\partial(\Delta_h\Delta_f-\Sigma_h\Sigma_f)}{\partial\theta_f} = -(1-\sigma)(1-\sigma-\xi) < 0 \text{ when } \sigma > 1$$

Derivation 4 (Optimal monetary rules). The optimal monetary policy rules (48) and (49) are derived by having the monetary authority maximize the equilibrium utility of a representative agent in its own country with respect to its money growth rate. Below is the solution for the problem of the Home monetary authority, but the Foreign monetary authority's problem is symmetric.

$$\max_{x} V(x, x^*) = \max_{x} \frac{\left([(1-\theta_h)n]^{1-\theta_h} [\theta_f n^*]^{\theta_h}\right)^{1-\sigma} - 1}{1-\sigma} - \chi n^{\xi}$$

Taking the derivative of V with respect to x gives:

$$\frac{\partial V}{\partial x} = C^{-\sigma} \left[(1-\theta_h)^2 \left(\frac{C^f}{C^h}\right)^{\theta_h} \frac{\partial n}{\partial x} + \theta_h \theta_f \left(\frac{C^h}{C^f}\right)^{1-\theta_h} \frac{\partial n^*}{\partial x} \right] - \chi \xi n^{\xi-1} \frac{\partial n}{\partial x}$$

where n, n^*, C^h, C^f, and C are given by (44), (45), (38), (39), and (**??**), respectively. Setting the derivative equal to zero, it can be rewritten:

$$(1-\theta_h)^{\xi} \left(C^h\right)^{\Delta_h} \left(C^f\right)^{\Sigma_h} \left[(1-\theta_h) + \theta_h \frac{n \frac{\partial n^*}{\partial x}}{n^* \frac{\partial n}{\partial x}} \right] = \chi \xi$$

where $\Delta_h = (1-\theta_h)(1-\sigma) - \xi$ and $\Sigma_h = \theta_h (1-\sigma)$. Writing C^h and C^f in terms of n and n^*, the expression can be rewritten in the following way:

$$(n)^{\Delta_h} (n^*)^{\Sigma_h} \left[(1-\theta_h) + \theta_h \frac{n \frac{\partial n^*}{\partial x}}{n^* \frac{\partial n}{\partial x}} \right] = \frac{\chi \xi}{(1-\theta_h)^{(1-\theta_h)(1-\sigma)} (\theta_f)^{\theta_h (1-\sigma)}}$$

The following two expressions are important for finding the solution and for understanding why the optimal monetary policy rules are independent of the policy choice of the other country.

$$\frac{n \frac{\partial n^*}{\partial x}}{n^* \frac{\partial n}{\partial x}} = \frac{-\Sigma_f}{\Delta_f} \quad \text{and} \quad (n)^{\Delta_h} (n^*)^{\Sigma_h} = x \Omega_h$$

where $\Delta_f = (1-\theta_f)(1-\sigma) - \xi$, $\Sigma_f = \theta_f (1-\sigma)$, and:

$$\Omega_h = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{\chi \xi}{(1-\theta_h)^{(1-\theta_h)(1-\sigma)} (\theta_f)^{\theta_h (1-\sigma)}}$$

So now the optimal money growth rate for the Home country can be written in the following way:

$$\begin{aligned} \hat{x} &= \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\Delta_f}{(1-\theta_h)\Delta_f - \theta_h \Sigma_f} \\ &= \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{(1-\theta_f)(1-\sigma) - \xi}{(1-\theta_h-\theta_f)(1-\sigma) - (1-\theta_h)\xi} \end{aligned}$$

Using the analogous symmetric value function of the Foreign representative agent, one can solve the Foreign monetary authority's maximization problem and get the symmetric result that the optimal rate of Foreign money growth is given by:

$$\begin{aligned}\hat{x}^* &= \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\Delta_h}{(1-\theta_f)\Delta_h - \theta_f \Sigma_h} \\ &= \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{(1-\theta_h)(1-\sigma) - \xi}{(1-\theta_h-\theta_f)(1-\sigma) - (1-\theta_f)\xi}\end{aligned}$$

The reason why $\hat{x}$ is not a function of x^* and why $\hat{x}^*$ is not a function of x is because the equilibrium derivative $\frac{\partial U(C)}{\partial x}$ divided by the equilibrium derivative $\frac{\partial g(n)}{\partial x}$ is independent of x^*. This reduces down to the two expressions above for $(n)^{\Delta_h}(n^*)^{\Sigma_h}$ and $\frac{n}{n^*}\frac{\partial n^*}{\partial x}\left(\frac{\partial n}{\partial x}\right)^{-1}$. Both of them are independent of x^*.

T-3 Properties of International Model CES aggregator

In this paper, I assume a specific case of the general CES functional form for aggregate consumption of a given Home or Foreign consumer. As defined in Section 2.2, the aggregate consumption levels of the differentiated goods of the Home and Foreign countries are, respectively:

$$C^h_{t+1} \equiv \left(\int_0^1 c_{t+1}(z)^{\frac{\varepsilon-1}{\varepsilon}}\,dz\right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall\, t \tag{T.3.1}$$

$$C^f_{t+1} \equiv \left(\int_0^1 c_{t+1}(z^*)^{\frac{\varepsilon-1}{\varepsilon}}\,dz^*\right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall\, t \tag{T.3.2}$$

where ε is the constant elasticity of substitution among differentiated goods in either the Home or Foreign country. The aggregator over both Home aggregate consumption C^h_{t+1} and aggregate Foreign consumption C^f_{t+1} takes the same general CES form as in (T.3.1) and (T.3.2).

$$C_{t+1} \equiv \left[(1-\theta)^{\frac{1}{\rho}}\left(C^h_{t+1}\right)^{\frac{\rho-1}{\rho}} + \theta^{\frac{1}{\rho}}\left(C^f_{t+1}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \quad \text{for} \quad \theta \in \left[0, \frac{1}{2}\right] \tag{T.3.3}$$

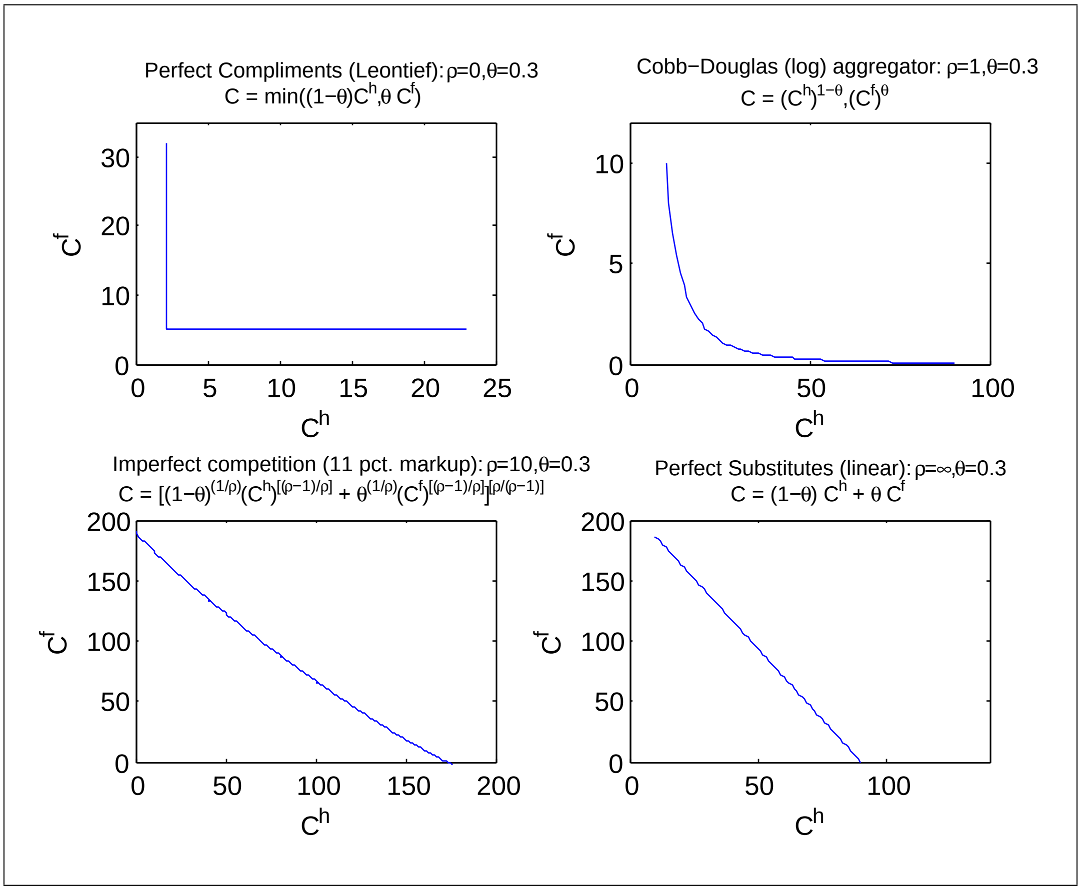
where θ is a Home-bias parameter and $\rho \geq 0$ is the elasticity of substitution between a unit of Home consumption and a unit of Foreign consumption. The only restriction is that the elasticity of substitution between aggregate Home consumption and aggregate Foreign consumption is assumed to be less than or equal to the elasticity of substitution among the differentiated goods in either country ($\rho \leq \varepsilon$). In the analyses in Section 2, I assume a specific case of (T.3.3) in which the aggregator assumes a Cobb-Douglas form ($\rho = 1$). The general CES aggregator is an attractive form because it nests so many economically relevant cases.

Figure 3 shows various specifications of the general CES aggregator function in (T.3.3). Taking the limit of (T.3.3) as $\rho \rightarrow 0$, a fixed level of aggregate consumption takes the Leontief form of perfect complements as shown in the first panel of Figure 3. Using L'Hospital's rule when taking the limit of (T.3.3) as $\rho \rightarrow 1$, the aggregator function corresponding to unit elasticity ($\rho = 1$) is Cobb-Douglas or log utility as shown in the second panel in Figure 3. Lastly, the fourth panel shows that the linear aggregator or perfect substitutes is the resulting aggregator function as $\rho \rightarrow \infty$. This reflects the case of perfect competition. Included in the third panel of Figure 3 shows the shape of the general CES aggregator function when the elasticity of substitution is at its often calibrated value of 10.

The key result here is that each constant consumption aggregator curve becomes flatter as the elasticity of substitution increases from the perfectly inelastic case of $\rho = 0$ to the perfectly elastic case of $\rho = \infty$. The exponent of $1/\rho$ on the Home bias terms is merely a convenience to make the resulting constant expenditure ratio a more simple expression.

It is important, however, to recognize that the common assumption of a logarithmic or Cobb-Douglas aggregator is implicitly assuming a unit elasticity of substitution

Figure 3: Various specifications of general CES aggregator function



between Home and Foreign aggregate consumption. As is shown in equation (19) of Section 2.2, the case of $\rho = 1$ implicitly makes the strong assumption that individuals exchange a constant share of their revenues for Foreign currency. Appendix T-4 addresses the solution to the model when the total consumption aggregator takes its general form ($\rho \geq 0$).

T-4 Solutions for General CES Aggregator

The purpose of this appendix is to document some of the solutions to the equilibrium problem when the CES aggregator over Home aggregate consumption is not restricted to the Cobb-Douglas case of unit elasticity of substitution ($\rho \neq 1$).

(T.4.1)
$$C_{t+1} \equiv \left[(1-\theta_h)^{\frac{1}{\rho}} \left(C^h_{t+1}\right)^{\frac{\rho-1}{\rho}} + \theta_h^{\frac{1}{\rho}} \left(C^f_{t+1}\right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \quad \text{for} \quad \theta_h \in \left(0, \frac{1}{2}\right]$$

The maximization problem analogous to (18) is the following:

(T.4.2)
$$\max_{m^f_t, p_t(z)} \frac{\left(\left[(1-\theta_h)^{\frac{1}{\rho}} \left(\left[\frac{p_t(z)}{P^h_t}\right]^{1-\varepsilon} \frac{xM_{t-1}}{P^h_{t+1}} - \frac{e_t m^f_t - (x-1)xM_{t-1}}{P^h_{t+1}} \right)^{\frac{\rho-1}{\rho}} + \theta_h^{\frac{1}{\rho}} \left(\frac{m^f_t}{P^f_{t+1}}\right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \right)^{1-\sigma} - 1}{1-\sigma} \ldots$$
$$-\chi \left[\left(\frac{p_t(z)}{P^h_t}\right)^{-\varepsilon} \frac{xM_{t-1}}{P^h_t} \right]^{\xi}$$

where the two first order conditions, analogous to (19) and (20), are:

(T.4.3)
$$\frac{P^h_{t+1}}{e_t P^f_{t+1}} \left(\frac{C^h_{t+1}}{C^f_{t+1}}\right)^{\frac{1}{\rho}} = \left(\frac{1-\theta_h}{\theta_h}\right)^{\frac{1}{\rho}}$$

(T.4.4)
$$(1-\theta_h)^{\frac{1}{\rho}} \left(\frac{\varepsilon-1}{\varepsilon}\right) \frac{p_t(z)}{P^h_{t+1}} \left(C_{t+1}\right)^{\frac{1}{\rho}-\sigma} \left(C^h_{t+1}\right)^{-\frac{1}{\rho}} = \chi\xi \left(n_t(z)\right)^{\xi-1}$$

where equation (T.4.3) equates the marginal cost of giving up a Home-currency unit of Home consumption for the marginal benefit of a Home-currency unit of Foreign consumption. Equation (T.4.4) equates the marginal benefit of raising price in terms of less demand and less disutility of labor to its marginal cost in terms of lost income in the next period of life. The market clearing conditions are the same as in Section 2.3. The equations that characterize an equilibrium in this case, given monetary policy (x, x^*) are shown in Table 3.

The steady-state equilibrium inflation rates are again equal to the money growth rates as in (36). However, because the first order condition for m^f_t in (T.4.3) no longer implies a constant expenditure share on Home and Foreign consumption, individuals can substitute away from Foreign expenditure when the inflation tax of the Foreign country's monetary policy adversely affects them. A key point here is that, when $\rho = 1$ and the aggregator is Cobb-Douglas, agents are bound to hold a specific fraction of their revenues in Foreign currency. Thus, $\rho = 1$ renders the demand for Foreign currency inelastic. When $\rho \neq 1$ the elasticity of demand for Foreign currency becomes elastic.

Table 3: Equilibrium conditions given x and x^* with general CES aggregator

	Home country	Foreign country
(19′)	$\frac{P^h_{t+1}}{e_t P^f_{t+1}}\left(\frac{C^h_{t+1}}{C^f_{t+1}}\right)^{\frac{1}{\rho}} = \left(\frac{1-\theta_h}{\theta_h}\right)^{\frac{1}{\rho}}$	$\frac{e_t P^f_{t+1}}{P^h_{t+1}}\left(\frac{C^{f*}_{t+1}}{C^{h*}_{t+1}}\right)^{\frac{1}{\rho}} = \left(\frac{1-\theta_f}{\theta_f}\right)^{\frac{1}{\rho}}$
(20′)	$(1-\theta_h)^{\frac{1}{\rho}}\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{p_t(z)}{P^h_{t+1}}\frac{\left(C_{t+1}\right)^{\frac{1}{\rho}-\sigma}}{\left(C^h_{t+1}\right)^{\frac{1}{\rho}}} = \chi\xi\left(n_t(z)\right)^{\xi-1}$	$(1-\theta_f)^{\frac{1}{\rho}}\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{p_t(z^*)}{P^f_{t+1}}\frac{\left(C^*_{t+1}\right)^{\frac{1}{\rho}-\sigma}}{\left(C^{f*}_{t+1}\right)^{\frac{1}{\rho}}} = \chi\xi\left(n_t(z^*)\right)^{\xi-1}$
(15)	$n_t(z) = \left(\frac{p_t(z)}{P^h_t}\right)^{-\varepsilon}\frac{xM_{t-1}}{P^h_t}$	$n_t(z^*) = \left(\frac{p_t(z^*)}{P^f_t}\right)^{-\varepsilon}\frac{x^*M^*_{t-1}}{P^f_t}$
(11)	$p_t(z)n_t(z) = m^h_t + e_t m^f_t$	$p_t(z^*)n_t(z^*) = m^{f*}_t + \frac{m^{h*}_t}{e_t}$
(16)	$C^h_{t+1} = \frac{m^h_t+(x-1)xM_{t-1}}{P^h_{t+1}}$	$C^{f*}_{t+1} = \frac{m^{f*}_t+(x^*-1)x^*M^*_{t-1}}{P^f_{t+1}}$
(17)	$C^f_{t+1} = \frac{m^f_t}{P^f_{t+1}}$	$C^{h*}_{t+1} = \frac{m^{h*}_t}{P^h_{t+1}}$
(??)	$C_{t+1} = \left[(1-\theta_h)^{\frac{1}{\rho}}\left(C^h_{t+1}\right)^{\frac{\rho-1}{\rho}} + \theta_h^{\frac{1}{\rho}}\left(C^f_{t+1}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$	$C^*_{t+1} = \left[(1-\theta_f)^{\frac{1}{\rho}}\left(C^{f*}_{t+1}\right)^{\frac{\rho-1}{\rho}} + \theta_f^{\frac{1}{\rho}}\left(C^{h*}_{t+1}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$
	Market clearing conditions	
(22)	$n_t(z) = c_t(z) + c^*_t(z)$	
(23)	$n_t(z^*) = c_t(z^*) + c^*_t(z^*)$	
(24)	$M_t = m^h_t + m^{h*}_t$	
(25)	$M^*_t = m^f_t + m^{f*}_t$	
(26)	$e_t m^f_t = m^{h*}_t$	

As in Section 2.4, the steady state equilibrium inflation level is found by substituting the money market clearing conditions (24) and (25) and the currency exchange market clearing condition (26) into the portfolio constraint (11) and its Foreign analogue, and then iterating the constraint one period forward.

(T.4.5) $$\frac{P^h_{t+1}}{P^h_t} = x$$

(T.4.6) $$\frac{P^f_{t+1}}{P^f_t} = x^*$$

The steady state equilibrium exchange rate is found by plugging the expressions for the currency shares from (28) and (29) into the currency exchange market clearing condition (26).

(T.4.7) $$e_t = \frac{(1-\phi)M_t}{(1-\phi^*)M^*_t}$$

Plugging in the expressions for the currency shares from (28) and (29), the equilibrium inflation rates (T.4.5) and (T.4.6), and using the currency exchange market clearing condition (26), the expressions for steady-state equilibrium aggregate consumption levels given x and x^* are the following:

$$C^h = \frac{(\phi + x - 1)n}{x} \tag{T.4.8}$$

$$C^f = \frac{(1-\phi^*)n^*}{x^*} \tag{T.4.9}$$

$$C^{f*} = \frac{(\phi^* + x^* - 1)n^*}{x^*} \tag{T.4.10}$$

$$C^{h^*} = \frac{(1-\phi)n}{x} \tag{T.4.11}$$

and the steady state equilibrium expressions for Home and Foreign employment are:

$$n = \frac{xM_{t-1}}{p_t(z)} \tag{T.4.12}$$

$$n^* = \frac{x^* M^*_{t-1}}{p_t(z^*)} \tag{T.4.13}$$

Now taking the steady state equilibrium values of (T.4.8) through (T.4.13) as well as the equilibrium characterizations for prices and the exchange rate from (T.4.5), (T.4.6), and (T.4.7), and substituting them into the two first order conditions for the Home country (T.4.3) and (T.4.4) and their two Foreign analogues, the steady state equilibrium is characterized by the following set of four equations in four unknowns $(\phi, p_t(z), \phi^*, p_t(z^*))$:

$$C^h \left(C^f\right)^{\rho-1} \left(C^{h*}\right)^{-\rho} = \frac{1-\theta_h}{\theta_h} \tag{T.4.14}$$

$$C^{f*} \left(C^{h*}\right)^{\rho-1} \left(C^f\right)^{-\rho} = \frac{1-\theta_f}{\theta_f} \tag{T.4.15}$$

$$\frac{(1-\theta_h)^{\frac{1}{\rho}}}{x} \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\left(C\right)^{\frac{1}{\rho}-\sigma}}{\left(C^h\right)^{\frac{1}{\rho}}} = \chi\xi \left(\frac{xM_{t-1}}{p_t(z)}\right)^{\xi-1} \tag{T.4.16}$$

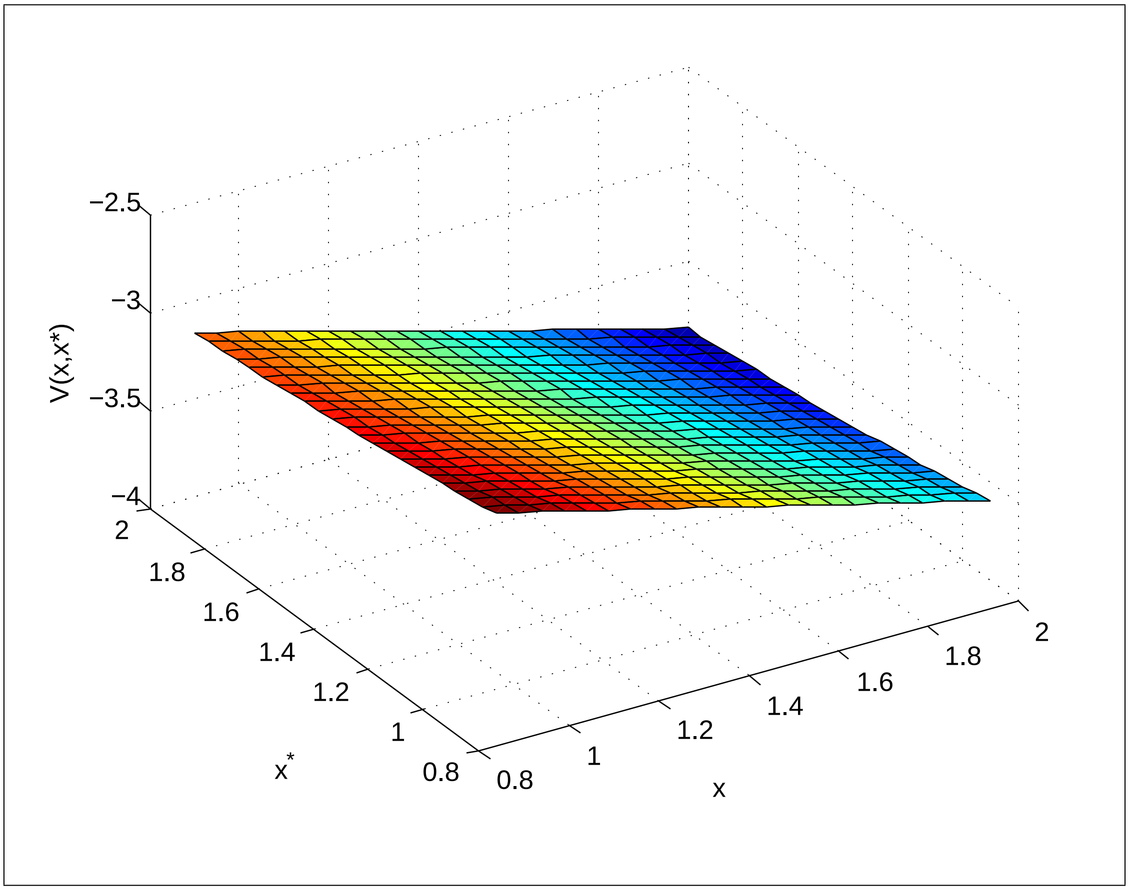
$$\frac{(1-\theta_f)^{\frac{1}{\rho}}}{x^*} \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\left(C^*\right)^{\frac{1}{\rho}-\sigma}}{\left(C^{f*}\right)^{\frac{1}{\rho}}} = \chi\xi \left(\frac{x^* M^*_{t-1}}{p_t(z^*)}\right)^{\xi-1} \tag{T.4.17}$$

Thus, the policy functions are functions of the parameters, x and x^*, and state variables M_{t-1} and M^*_{t-1}. The state variables are normalized to 1 in this case without loss of generality.

Because the model with the general CES aggregator ($0 < \rho < \infty$ and $\rho \neq 1$) has no analytical solution, I solve it numerically.[17] Figure 4 shows the value function $V(x, x^*$ of a representative agent in the Home country. It was calibrated such that $\theta = 0.75$, $\sigma = 3$, $\varepsilon = 10$, $\chi = 2$, $\xi = 2$, $\rho = 2$, and $x, x^* \in (0.9, 2.0)$.

[17] I discretize the (x, x^*) state space and use a Nelder-Mead simplex search method to find the solution at each point. The code for this computation is available upon request.

Figure 4: $V(x, x^*)$ in general CES case



Calibrated parameters: $\theta_h = \theta_f = 0.75$, $\sigma = 3$, $\varepsilon = 10$, $\chi = 2$, $\xi = 2$, $\rho = 0.95$.

The main difference here from the Cobb-Douglas aggregator case in the paper in which $\rho = 1$ is that the optimal expenditure share on Home currency ϕ is now a function of both x and x^*. So individuals can substitute away from Foreign currency expenditure if it becomes too expensive in terms of Home consumption. This induces an international Bertrand duopoly situation between the two monetary authorities. That is, the lower money growth rate a monetary authority chooses, the more attractive are the terms of trade for a Foreign country. It becomes a race to the bottom and the world equilibrium monetary policy is $(x = x* = 0)$.

T-5 Frictions

Before moving on to the results from Section 2.4, it is instructive to highlight the two frictions present in this model—money and imperfect competition—and their interplay with the level of openness. The two frictions are most easily isolated in a closed economy when the other friction is shut down. The inefficiencies caused by these two frictions are manifested in this setting as the "labor wedge" outlined in Chari, Kehoe, and McGrattan (2007).[18] The efficient allocation is found by solving the planner's problem of maximizing the utility of the period-t old from consumption minus the disutility of labor of the period-t young in the closed economy case $\theta_h = 0$, subject to the resource constraint.

$$\begin{aligned} &\max_{C_t^h, n_t} u\left(C_t^h\right) - g\left(n_t\right) \\ &\quad \text{s.t.} \quad C_t^h = n_t \end{aligned} \tag{T.5.18}$$

The planner's solution steady state equilibrium is the following:

$$(C_{ps}, n_{ps}) : \quad u'(C^h) = g'(n) \tag{T.5.19}$$

The deviation from the planner's solution created by the presence of imperfect competition is isolated by looking at the closed economy decentralized steady state solution where $\theta_h = 0$ in which the money growth rate is fixed at $x = 1$. The first order condition in (20) can be written as:

$$(C_{ic}, n_{ic}) : \quad u'(C) = \Phi g'(n) \tag{T.5.20}$$

where $\Phi = \frac{\varepsilon}{\varepsilon - 1} \geq 1$ and (T.5.20) represents that marginal utility of consumption equals a markup over marginal cost. The monopoly power enjoyed by firms resulting from the imperfect substitutability ε of their goods allows producers to raise prices above the efficient level and lower output in order to maximize profits. Thus, $(C_{ic}, n_{ic}) \ll (C_{ps}, n_{ps})$, and (C_{ic}, n_{ic}) decreases as the degree imperfect competition Φ increases (as ε decreases).

In like manner, the deviation from the planner's solution created by the money growth rate is isolated by looking at the closed economy decentralized steady state solution where $\theta_h = 0$ in which producers are perfectly competitive $\varepsilon = \infty$ ($\Phi = 1$). The first order condition in (20) can now be written as:

$$(C_{mp}, n_{mp}) : \quad \frac{1}{x} u'(C) = g'(n) \tag{T.5.21}$$

Equation (T.5.21) highlights the reason why expansionary monetary policy is thought of as an inflation tax. For higher money growth rates, the marginal benefit of an extra

[18] However, a key point on which this paper differs from Chari, Kehoe, and McGrattan (2007) is that money is set optimally in this paper and not stochastic. But Chari, Kehoe, and McGrattan (2007) do conclude that the labor-wedge channel does explain much of the observed variation in business cycles.

unit of labor decreases. Another way of looking at this problem is that the marginal productivity of labor is equal to 1, given the linear production technology. But the real wage in the closed economy is $\frac{1}{x}$. So for any money growth rate greater than 1, the real wage is less than the marginal productivity of labor. The result is that labor supplied is inefficiently low and $(C_{mp}, n_{mp}) \ll (C_{ps}, n_{ps})$ for all $x > 1$. Conversely, $(C_{mp}, n_{mp}) \gg (C_{ps}, n_{ps})$ for all $x < 1$. If the money growth rate is set optimally, the first best policy is $x = 1$ in this closed economy setting.

The interplay between openness, monetary policy and imperfect competition is seen when the closed economy frictions described preceding paragraphs are compared to their open economy counterparts. In the closed economy above, any money growth rate greater than the inverse of the markup gives a leisure subsidy that is dominated by an inflation tax, both of which are borne entirely by the agents of the closed country. However, when the two countries are open $(\theta_h, \theta_f > 0)$, the inflation tax imposed by increasing the money growth rate is no longer borne solely by Home agents. Furthermore, increased money growth by the Home monetary authority actually increases the real wage through the terms of trade appreciation and increased preference weight on Foreign consumption. This added benefit of Home money growth is due to the international monopoly power of the Home monetary authority derived from the degree of inelastic demand fo imports by Foreign consumers.[19]

From the expressions for Home and Foreign employment in (44) and (45), the Home leisure subsidy results from the negative effect of an increase in x and the Foreign leisure tax results from the positive effect of an increase in x. The consumption tax of inflation can be seen by taking the derivative of equilibrium Home aggregate consumption C and Foreign aggregate consumption with respect to x.

(T.5.22)

$$C = \left[(1-\theta_h)n\right]^{1-\theta_h}\left[\theta_f n^*\right]^{\theta_h}$$

$$= (1-\theta_h)^{1-\theta_h}\theta_f^{\theta_h}\left[\frac{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}\theta_f^{\theta_h(1-\sigma)}}{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}\theta_h^{\theta_f(1-\sigma)}}\right]^{\frac{\theta_h(1-\sigma-\xi)}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}(x)^{\frac{(1-\theta_h)\Delta_f-\theta_h\Sigma_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}(x^*)^{\frac{\theta_h\Delta_h-(1-\theta_h)\Sigma_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}$$

(T.5.23)

$$C^* = \left[(1-\theta_f)n^*\right]^{1-\theta_f}\left[\theta_h n\right]^{\theta_f}$$

$$= (1-\theta_f)^{1-\theta_f}\theta_h^{\theta_f}\left[\frac{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}\theta_h^{\theta_f(1-\sigma)}}{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}\theta_f^{\theta_h(1-\sigma)}}\right]^{\frac{\theta_f(1-\sigma-\xi)}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}(x^*)^{\frac{(1-\theta_f)\Delta_h-\theta_f\Sigma_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}(x)^{\frac{\theta_f\Delta_f-(1-\theta_f)\Sigma_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}$$

The exponents on x in both (T.5.22) and (T.5.23) are both negative, but the exponent on x in (T.5.22) is larger in absolute value. That is, an increase in the Home money growth rate will cause a decrease in both the Home aggregate consumption C and Foreign aggregate consumption C^*, but the decrease in C is greater than the decrease in C^*. This latter fact is seen more clearly when steady state equilibrium relative

[19] Recall that the constant expenditure share principle derives from the first order condition of the utility with the Cobb-Douglas aggregate consumption.

aggregate consumption is expressed as follows:

$$\text{(T.5.24)} \qquad \frac{C}{C^*} = \frac{(1-\theta_h)^{(1-\theta_h)}\theta_f^{\theta_h}}{(1-\theta_f)^{(1-\theta_f)}\theta_h^{\theta_f}} \left[\frac{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}\theta_h^{\theta_f(1-\sigma)}x}{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}\theta_f^{\theta_h(1-\sigma)}x^*}\right]^{\frac{(1-\theta_h-\theta_f)(1-\sigma-\xi)}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}$$

The exponent on the bracketed term is negative, so an increase in x makes C decrease more than C^*. Thus, the inflation tax in the open economy is not just a decrease in equilibrium Home aggregate consumption C as in the closed economy case, but also a decrease in Foreign aggregate consumption C^* and an increase in Foreign employment n^*.

As was mentioned earlier, the Home leisure subsidy is the only benefit of inflation in the open economy that also exists in the closed economy. However, in contrast to the decrease in the real wage in a closed economy, an increase in the Home money growth rate x increases the real wage in the open economy setting. The real wage in the open economy is the extra aggregate consumption from an extra unit of labor. Thus, the Home real wage is the derivative of Home aggregate consumption C with respect to n.

$$\text{(T.5.25)} \qquad \begin{aligned} \frac{\partial C}{\partial n} &= (1-\theta_h)^{2-\theta_h}\theta_f^{\theta_h}\left(\frac{n^*}{n}\right)^{\theta_h} \\ &= (1-\theta_h)^{2-\theta_h}\theta_f^{\theta_h}\left[\frac{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}\theta_f^{\theta_h(1-\sigma)}x^*}{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}\theta_h^{\theta_f(1-\sigma)}x}\right]^{\frac{\theta_h(1-\sigma-\xi)}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \end{aligned}$$

Because the exponent on the bracketed term is negative, the effect of an increase in the Home money growth rate x is to increase the real wage. On the other hand, an increase in the Foreign money growth rate x^* is to decrease the real wage due to the positive effect of x^* on n.

This real-wage benefit of Home inflation is driven by two components. First, as has been documented by Corsetti and Pesenti (2001), Cooley and Quadrini (2003), Cooper and Kempf (2003), and Arseneau (2007), an increase in the Home money growth rate x causes the terms of trade to appreciate in favor of the Home country. The terms of trade for a given country is defined as the price of its exports in terms of its imports. In the steady state equilibrium, the terms of trade for the Home country can be expressed as follows:

$$\text{(T.5.26)} \qquad ToT \equiv \frac{P_{t+1}^h}{e_t P_{t+1}^f} = \frac{\theta_f}{\theta_h}\left[\frac{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}\theta_f^{\theta_h(1-\sigma)}x^*}{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}\theta_h^{\theta_f(1-\sigma)}x}\right]^{\frac{1-\sigma-\xi}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}$$

Again, because the exponent on the bracketed term is negative, the effect of an increase in the Home money growth rate x is to increase the cost of Home exports in terms of Home imports. On the other hand, an increase in the Foreign money growth rate x^* is to decrease the terms of trade. The second component of the real-wage benefit of Home inflation is simply that increased openness means that more weight is placed on Foreign consumption which is amplified by the terms-of-trade appreciation.

So the objective of the Home monetary authority is to set its money growth rate such that the benefits of the inflation caused by x (leisure subsidy and real-wage benefit) equal the costs (consumption tax). The real-wage benefit is a direct result of the monopoly power that the monetary authority enjoys in international markets. And this monopoly power derives from the degree of inelastic demand for Foreign goods, as shown in the first order condition for Foreign currency balances (19).

Lastly, looking at the expression for the optimal Home money growth rate x in (48), it is no surprise that as the degree of imperfect competition increases in the Home country, the country-specific welfare benefits that the monetary authority can obtain from increasing the money growth rate decrease. Intuitively, the monopoly rents from the imperfect competition structure replace the monopoly rents obtained by the monetary authority through increasing the money growth rate.