Openness, Inflation, and Imperfect Competition: Theory and Evidence *

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Abstract

In this paper, I propose a channel through which increased openness to international trade can increase equilibrium inflation in the opening country. I then perform an empirical test of the theory and find supporting evidence that openness can be inflationary. The inflationary effect of openness depends critically on the level of imperfect competition, both within countries and on the international market. Because domestic and foreign goods are imperfect substitutes, a monetary authority can export some of the inflation tax of money growth to foreign holders of their currency. However, this inflationary incentive is dampened by the degree of imperfect competition within the country. An empirical test of the effect of openness on inflation that controls for the degree of imperfect competition within each country provides evidence of a positive relationship. This finding runs contrary to the broadest vein of the literature addressing the question of openness and inflation.

keywords: Optimal Monetary Policy; Imperfect Competition; International Monetary Policy

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1 Introduction

The nebulous term "globalization" is most often used to describe the increased economic integration of countries in the last 30 years. In making that term more specific, many branches of the international economics literature have studied questions about the effects of a countries openness on various economic outcomes. The goal of this paper is to provide evidence on the effect of openness on inflation.

First, I provide a theoretical model that illustrates a channel through which increase openness of a country to international trade can increase equilibrium inflation in that country. This positive effect of openness on inflation depends critically on the level of imperfect competition, both within countries and on the international market. Because domestic and foreign goods are imperfect substitutes, a monetary authority can export some of the inflation tax of money growth to foreign holders of their currency. In effect, a monetary authority enjoys a degree of monopoly power in international markets due to foreign holders of the domestic currency and some degree of inelasticity of substitution between domestic and foreign consumption.

However, this inflationary incentive is dampened by the degree of imperfect competition within the country. The intuition is that some rents exist in the international marketplace, and that the private holders of monopoly power within a country will obtain as much of those rents as possible given the level of imperfect competition in that country. And whatever portion of those rents are left over is taken by the monetary authority through its policy instrument.

In Section 2, I present the model. It is a two-country overlapping generations (OLG) model with imperfectly competitive labor markets, a monetary authority in each country that chooses a money growth rate to maximize the welfare of its citizens, and in which domestic and foreign consumption are imperfect substitutes. This model draws from Cooper and Kempf (2003) who use a two-country OLG model with perfectly competitive markets within each country, optimal monetary authorities, and imperfectly substitutable domestic and foreign goods to answer the question of when currency unions are optimal. However, one implicit result of their work is that increased openness is inflationary.

I follow the method of modeling imperfect competition within each country from Arseneau (2007) who uses a two-country infinite horizon model with imperfectly competitive goods markets, optimal monetary policy, and imperfectly substitutable domestic and foreign consumption goods to study the effects of imperfect competition on inflation in a multi-country environment. He finds that the increased imperfect competition (less competition) has a negative effect on equilibrium inflation rates. However, rather than model the degree of openness of a country, Arseneau (2007) models the size of a country. Corsetti and Pesenti (2001) use a similar model with stochastic money growth.

However, none of the above papers directly study the question of openness and inflation.[1] The fundamental theoretical underpinnings of this question come from Rogoff (1985), who uses a two-country adaptation of Barro and Gordon (1983) to illustrate a channel through which increased openness decreases equilibrium inflation. In a closed economy Barro-Gordon setting, the well known time-consistency problem generates a situation of suboptimally high inflation and employment and output at their natural rates. However, when the country becomes open to international trade, the time-consistent inflationary incentive of the monetary authority decreases because of its adverse effect on foreign demand for domestically produced goods.

The channel that I model is different from Rogoff (1985) in two major ways. First, I model optimal monetary policy in the simplest possible way—commitment to a constant money growth rate in order to maximize country welfare. This is merely a simplifying assumption instead of time-consistent monetary policy because the goal is not the effect of monetary policy but rather the effect of openness on inflation in the presence of optimal monetary policy. The second major difference is the structure of demand on international markets. Rogoff (1985) implicitly assumes that domestic and foreign goods are perfect substitutes, whereas I allow some degree of inelasticity of substitution between home and foreign goods. This is the key characteristic that

[1] Evans (2007) and Wynne and Kersting (2007) provide summaries of the literature on openness and inflation.

generates the *beggar-thy-neighbor* incentive for a monetary authority to inflate in an international setting.[2]

In Section 3, I present some empirical tests of the effect of openness on inflation. The empirical methods of this paper follow Romer (1993). He tested the theoretical prediction of Rogoff (1985), by running regressions of a proxy for openness (average import share of GDP) on the average inflation rate for a sample of 114 countries over the period from 1973 to 1987. Across a number of different specifications, Romer found a robust negative relationship between openness and inflation. His result was further confirmed in follow-up studies by Lane (1997) and Terra (1998).

The striking result that comes from the empirical estimates in this paper is that the sign of the effect of openness on inflation becomes positive when looking at a the later sample period from 1988 to 2002. Another innovation of this paper in the empirical openness and inflation literature is that I control for the level of imperfect competition within each country. Using limited manufacturing and nonmanufacturing markup data from Høj, Jimenez, Maher, Nicoletti, and Wise (2007) and using more broadly available union membership rates and union coverage rates data, I include these proxies for countrywide imperfect competition levels as controls in the regression.

I find that the empirical tests in this paper provide supporting evidence of the prediction of my theoretical model that increased openness can result in increased inflation. This is not to make a normative statement that openness to international trade is negative. Rather, the findings of this paper illustrate in more detail the costs and benefits to openness to international trade.

2 Model

The model in this section uses a two-country overlapping generations framework similar to Cooper and Kempf (2003). However, I add imperfectly competitive labor markets within each country following Arseneau (2007) and Corsetti and Pesenti (2001).

[2] This international market power story fits well with the current situation in which the dollar is the most widely held currency in the world, and that this monopoly power allows the United States to maintain a large budget deficit.

However, the imperfect competition in this model is in the labor market rather than the goods producing market. Each country has a continuum of perfectly competitive firms that produce by hiring the differentiated labor at a contracted wage. Each country also has a monetary authority that commits to a constant money growth rate at the beginning of time in order to maximize the welfare of the citizens of its own country.

2.1 Money

The optimal monetary policy structure of this model is the simplest possible form. Money is held because it is the only store of value, and I assume that the monetary authority must commit to a constant money growth rate at the beginning of time. Money in this model is not neutral because of a price rigidity arising from wage contracts. The consumption tax levied by money growth in a steady state equilibrium reflects the expected increased deterioration of purchasing power over time on the part of consumers.

The objective of the monetary authority in each country, which will be made more explicit in Section 2.5, is to choose a fixed (gross) money growth rate $x_t = x$ or $x_t^* = x^*$ at the beginning of time in such a way as to maximize the welfare of its own citizens. I assume here that the monetary authority is committed to its money growth rate and cannot deviate once it has chosen its money growth path.[3] Money is held because it is the only store of value, and monetary policy is not neutral in the steady state because it drives a wedge between the marginal rate of substitution and the real wage.[4]

Let M_t and M_t^* be the aggregate supply of Home currency and Foreign currency, respectively, in period t. I normalize the initial supply of Home and Foreign currency

[3]This is the simplest possible optimal monetary policy structure. The reason to avoid discretionary monetary policy in this paper is due to the resulting characteristic of multiple equilibria, most of which are unstable sunspot equilibria characterized by expectations traps. King and Wolman (2004) is a good reference on multiple equilibria in models of discretionary monetary policy, which builds on the seminal work of Kydland and Prescott (1977) and Barro and Gordon (1983). Also, see Chatterjee, Cooper, and Ravikumar (1993).

[4]Intuitively, increased money growth taxes the purchasing power of the only store of value, which causes agents to take more leisure in the first period of their lives.

to 1 and divide it equally among the period-1 consumers at the beginning of the period.

$$M_0 = M_0^* = 1 \quad \text{and} \quad m_0^h = m_0^f = m_0^{h*} = m_0^{f*} = \frac{1}{2} \tag{1}$$

The variables m_0^h and m_0^{f*} are the individual holdings of Home currency by Home consumers and Foreign currency by Foreign consumers, respectively, at the beginning of period 1. Each country's monetary authority makes non-proportional transfers of $(x-1)M_{t-1}^h$ to each Home consumer in period t and $(x^*-1)M_{t-1}^f$ to each Foreign consumer where x and x^* represent the respective constant gross money growth rates of each country. So aggregate supply of currency in each country obeys the following laws of motion.

$$M_{t+1} = xM_t \tag{2}$$

$$M_{t+1}^* = x^* M_t^* \tag{3}$$

This implies that the following relationships for τ_{t+1} and τ_{t+1}^* represent the non-proportional transfer to each Home consumer and to each Foreign consumer by their respective monetary authorities.

$$\tau_{t+1} = (x-1)\, M_t \tag{4}$$

$$\tau_{t+1}^* = (x^*-1)\, M_t^* \tag{5}$$

At the end of the first period of their lives, workers make a portfolio decision of how much of each type of currency to hold. They have just received wages represented by either $p_t(z)n_t(z)$ in Home currency or $p_t(z^*)n_t(z^*)$ in Foreign currency for their differentiated labor hired by the identical firms in their own country. Before the end of the first period of life, workers in each country exchange some of their wages in own-country currency balances for other-country currency balances at the exchange rate e_t as shown in the budget constraint equation (17). Let m_t^h and m_t^f represent each Home worker's portfolio choice between Home and Foreign currency, respectively, in period t. Because the monetary authority of each country only transfers currency to its own consumers, the laws of motion for individual currency balances are the following:

$$m^h_{t+1} = m^h_t + \tau_{t+1} \tag{6}$$

$$m^f_{t+1} = m^f_t \tag{7}$$

$$m^{f*}_{t+1} = m^{f*}_t + \tau^*_{t+1} \tag{8}$$

$$m^{h*}_{t+1} = m^{h*}_t \tag{9}$$

Because the equilibrium currency holdings within each country are symmetric, then $m^h_t, m^f_t, m^{f*}_t, m^{h*}_t$ represent the aggregate amounts of each currency ($M^h_t, M^f_t, M^{h*}_t, M^{f*}_t$) held in each country in each period.

2.2 Firms

The Home and Foreign country each have a unit measure of identical infinitely lived firms that produce a consumption good y_t and y^*_t, respectively, each period. Within each country, the firms are perfectly competitive as characterized by a zero profit condition and can be treated as a single representative firm. The elasticity of substitution between Home and Foreign goods is given by the parameter $\rho \geq 0$, which enters into individual preferences described in Section 2.3.[5]

The representative firm in each country produces a final good y_t using differentiated labor $n_t(z)$ from its own country, where $z \in [0, 1]$ indexes the type of labor. The

[5]For most of this paper, I assume that $\rho = 1$, which results in a Cobb-Douglas aggregator or utility function as shown in Equation 21. Technical Appendix T-3 (available upon request) shows the general case of CES preferences in which $\rho \geq 0$ and shows some of its limiting values such as Leontief preferences ($\rho = 0$), Cobb-Douglas preferences ($\rho = 1$), and perfect substitutes ($\rho = \infty$).

production technology takes the constant elasticity of substitution form:[6]

$$y_t \equiv \left(\int_0^1 n_t(z)^{\frac{\varepsilon-1}{\varepsilon}} \, dz \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall\, t \tag{10}$$

$$y_t^* \equiv \left(\int_0^1 n_t(z^*)^{\frac{\varepsilon-1}{\varepsilon}} \, dz^* \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall\, t \tag{11}$$

where $\varepsilon \geq 1$ represents the elasticity of substitution among all the differentiated types of labor available to the firm in a given country.[7]

Note here that the imperfect competition is not on the part of firms. Instead, the suppliers of labor will possess the market power. However, a similar model used by Evans (2007) places the imperfect competition on the part of differentiated intermediate goods producers, and the results do not change.

The representative firm maximizes profits π_t by choosing how much of each type of labor within its own country $n_t(z)$ to hire in order to produce y_t units of the final consumption good given a market selling price for the good P_t^h and the negotiated wages for each of the different types of labor $w_t(z)$. Substituting the production function from (10) into the profit equation for y_t, the firm's problem becomes:

$$\max_{n_t(z)} \; \pi_t = P_t^h \left(\int_0^1 n_t(z)^{\frac{\varepsilon-1}{\varepsilon}} \, dz \right)^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 w_t(z) n_t(z) \, dz \quad \forall\, t \tag{12}$$

The resulting function for labor demand is the following.[8]

$$n_t(z) = \left(\frac{w_t(z)}{P_t^h} \right)^{-\varepsilon} y_t \quad \forall\, t, z \tag{13}$$

[6]The CES production function was first introduced by Arrow, Chenery, Minhas, and Solow (1961) and was extended to the n-input case by Uzawa (1962) and McFadden (1963). The idea of the differentiated inputs resulting in imperfect competition was then formalized by Dixit and Stiglitz (1977). The difference here is that the good produced in each country is a composite of the differentiated inputs rather than aggregate consumption being a composite of differentiated goods consumption.

[7]This is analogous to a situation in which firms can hire CEOs, accountants, and janitors. Each is an imperfect substitute for the other. I have assumed that the elasticities of substitution among differentiated labor types in the two countries are equal $\varepsilon = \varepsilon^* \geq 0$. However, this assumption does not change the sign of the effect of openness on inflation. Furthermore, the elasticity must satisfy $\varepsilon \geq 1$ because any $\varepsilon \in [0, 1)$ implies a negative markup at which a firm would not produce.

[8]See Derivation 1 in Technical Appendix T-1 (available upon request).

The expression for the price level of the consumption good P_t^h is pinned down by the zero profit condition.[9]

$$P_t^h = \left(\int_0^1 w_t(z)^{1-\varepsilon}\, dz \right)^{\frac{1}{1-\varepsilon}} \quad \forall\, t \tag{14}$$

The expression for an aggregate Home-country consumer price level P_t as a function of the price of Home-produced consumption P_t^h and Foreign-produced consumption P_t^f takes the following form:

$$P_t = \frac{1}{(1-\theta^h)^{1-\theta^h} (\theta^h)^{\theta^h}} \left(P_t^h \right)^{1-\theta^h} \left(e_{t-1} P_t^f \right)^{\theta^h} \tag{15}$$

where e_t is the exchange rate.[10]

2.3 Individuals

A unit measure of agents is born in each period in both the Home country (indexed by z) and the Foreign country (indexed by z^*). Each generation of agents lives for two periods. They work in the first period, and consume in the second period. From this point forward, I will show the problem of a Home-country agent. But the problem of a Foreign country agent is symmetric.

In the first period of their lives, individuals can either enjoy leisure l_t or provide labor $n_t(z)$ subject to their endowment of one unit of time.

$$l_t + n_t(z) = 1 \quad \forall t, z \tag{16}$$

Labor is not mobile across countries. Labor markets are imperfectly competitive in this model because each worker's labor is differentiated from that of all the other workers, analogous to Dixit and Stiglitz's (1977) differentiated goods model. As was shown in the firm production functions in equations (10) and (11) each differentiated type of labor is imperfectly substitutable among the other types of labor available

[9]See Derivation 2 in Technical Appendix T-1 (available upon request).

[10]See equation (T.2.13) in Technical Appendix T-2 for derivation (available upon request).

within a given country for producing the homogeneous country-specific consumption good.

At the beginning of the first period of life, an individual knows the monetary policy of both the Home and Foreign country (x, x^*), the aggregate money supply in each country (M_t, M_t^*), the labor demand functions for own-country firms (13), and the resulting pricing equations in each country (14). Because individuals in the first period of life know the firm's labor demand function (13) and because these workers can set their wage contract at the beginning of the period, labor supply equals labor demand. That is, because changes in an individual's amount of labor supplied and contracted wage do not affect the amount of country-specific output and price level, respectively, the choice of wage level determines the labor supply amount.[11]

Once the wage $w_t(z)$ has been contracted between each worker and the firm, each worker supplies labor $n_t(z)$, and the representative firm produces the amount y_t of the country-specific consumption good according to the Dixit-Stiglitz CES production technology in (10). The firm then sells its output to consumers from both the Home and Foreign countries who have had their currency balances augmented by the non-proportional transfers from their respective monetary authorities. Firms then take these revenues in their own country's currency and pay their workers according to the contracted wage rate.[12] At the end of the period, workers take their earnings in their own-country currency and decide how much of it to trade for foreign currency balances:

$$w_t(z)n_t(z) = m_t^h + e_t m_t^f \tag{17}$$

where e_t is the exchange rate.

In the second period of life, the workers become consumers. They receive the non-proportional transfer from the monetary authority τ_{t+1} and they spend currency

[11] See Appendix A-1 for proof.

[12] These wage contracts are only considered a price friction in a stochastic or time-consistent monetary policy environment. Wage contracts are not a price friction in this perfect foresight, policy-commitment environment.

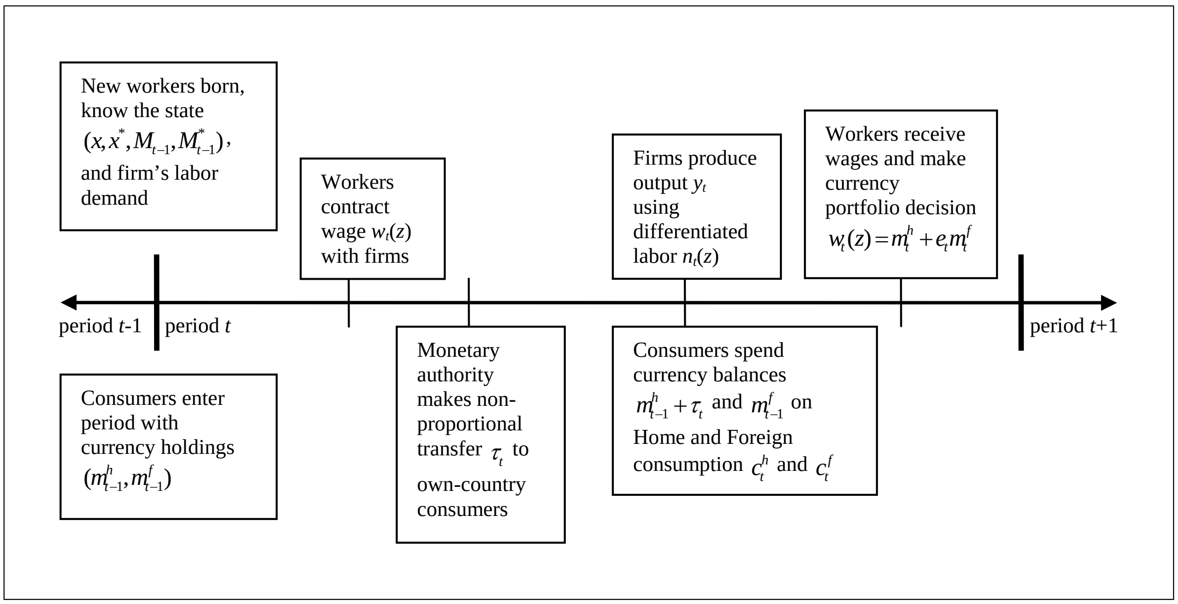
balances according to the following cash-in-advance constraints:

$$P_{t+1}^{h} c_{t+1}^{h} = m_t^h + \tau_{t+1} \tag{18}$$

$$P_{t+1}^{f} c_{t+1}^{f} = m_t^f \tag{19}$$

where c_{t+1}^h and c_{t+1}^f are the consumption of Home and Foreign goods, respectively, by a Home country agent. The prices of the Home and Foreign goods in terms of their respective currencies are P_{t+1}^h and P_{t+1}^f. The timing of this model is illustrated in Figure 1.

Figure 1: Timing of two-country OLG model



Lifetime utility of a Home-country agent is defined as an additively separable function over an aggregate consumption basket c_{t+1} and labor $n_t(z)$:

$$\begin{aligned} &U\left(c_{t+1}, n_t(z)\right) = u\left(c_{t+1}\right) - g\left(n_t(z)\right) \\ &\quad \text{where} \quad u\left(c_{t+1}\right) = \frac{\left(c_{t+1}\right)^{1-\sigma} - 1}{1-\sigma} \quad \text{for} \quad \sigma > 0 \\ &\quad \text{and} \quad g\left(n_t(z)\right) = \chi\left(n_t(z)\right)^{\xi} \quad \text{for} \quad \chi > 0 \quad \text{and} \quad \xi \geq 1 \end{aligned} \tag{20}$$

where σ represents the coefficient of relative risk aversion, and χ and ξ are the level and shape parameters, respectively, of the function for the disutility of labor. Individual aggregate consumption c_{t+1} is defined as a Cobb-Douglas constant elasticity of substitution aggregator over Home and Foreign consumption:

$$c_{t+1} \equiv \left(c^h_{t+1}\right)^{1-\theta_h} \left(c^f_{t+1}\right)^{\theta_h} \quad \forall t \quad \text{and} \quad \theta_h \in \left[0, \frac{1}{2}\right] \tag{21}$$

where c^h_{t+1} is individual consumption of the Home-produced good by the Home agent, c^f_{t+1} is individual consumption of the Foreign-produced good by the Home agent and θ^h is the home bias parameter. Because θ^h reflects the preference weighting that Home consumers place on Foreign consumption, it characterizes the degree of openness of the Home country.[13]

The Home consumer's demand for Home-produced and Foreign-produced consumption can be derived from the consumer's expenditure minimization problem in the second period of life, given the prices of Home-produced consumption P^h_{t+1} and Foreign-produced consumption P^f_{t+1} and given the level of aggregate consumption c_{t+1}.[14]

$$c^h_{t+1} = (1-\theta^h)\left(\frac{P^h_{t+1}}{P_{t+1}}\right)^{-1} c_{t+1} \tag{22}$$

$$c^f_{t+1} = \theta^h \left(\frac{e_t P^f_{t+1}}{P_{t+1}}\right)^{-1} c_{t+1} \tag{23}$$

Note that these consumption demand equations are analogous to the differentiated labor input demand equations in (13), except that they include the home-bias parameter. Also note that dividing (22) by (23) gives the constant expenditure share equation from the first order condition in (29). This simply because the demand equations from expenditure minimization and utility maximization are equivalent.

The objective of each individual in the Home country can be represented as a choice of a contracted wage rate $w_t(z)$ and a currency portfolio decision m^h_t and m^f_t

[13]Foreign consumers' degree of openness θ^f is not forced to be symmetric to the Home consumers' parameter. As will be shown later, θ^h is a good way to parameterize the degree of openness because it equals the import share in equilibrium, which has been a proxy for openness in many previous empirical studies (e.g., Romer (1993), Lane (1997), and Terra (1998)).

[14]See equations (T.2.8) and (T.2.9) in Technical Appendix T-2 for derivation (available upon request).

in order to maximize lifetime utility.

$$\max_{m_t^h, m_t^f, w_t(z)} \frac{\left[\left(c_{t+1}^h\right)^{1-\theta_h}\left(c_{t+1}^f\right)^{\theta_h}\right]^{1-\sigma} - 1}{1-\sigma} - \chi\left(n_t(z)\right)^{\xi} \tag{24}$$

$$\text{s.t.} \quad w_t(z)n_t(z) = m_t^h + e_t m_t^f \tag{17}$$

$$P_{t+1}^h c_{t+1}^h = m_t^h + \tau_{t+1} \tag{18}$$

$$P_{t+1}^f c_{t+1}^f = m_t^f \tag{19}$$

where (17) is the budget constraint reflecting the portfolio decision and (18) and (19) are the cash-in-advance constraints.

The two cash-in-advance constraints can be thought of as a simplification of one equilibrium outcome of a richer environment in which governments or monetary authorities strategically choose what currencies to accept for exchange that takes place within their borders. Matsuyama, Kiyotaki, and Matsui (1993) present a random matching search model of money after the flavor of Kiyotaki and Wright (1989) in which blocks of agents (countries) choose which currencies to accept for local and international transactions based on the likelihood of that currency being accepted in future transactions. In one equilibrium, corresponding to the two cash-in-advance constraint environment of this paper, each block of agents (country) only accepts local currency for all local and international transactions.

Another equilibrium in the Matsuyama, Kiyotaki, and Matsui (1993) is the case in which vendors in both countries accept currency of both countries. This is analogous to the more standard approach in the NOEM literature as exemplified by Corsetti and Pesenti (2001). Their environment is one characterized by a single cash-in-advance constraint in which producers sell their goods in both countries and charge a price in terms of Home currency and a price in terms of Foreign currency. The exchange rate is then pinned down by an assumption of the law of one price.

The reason for choosing the two cash-in-advance constraints approach as shown in equations (18) and (19) instead of the more standard Corsetti and Pesenti (2001) method of one cash-in-advance constraint and the law of one price is that the method employed here gives rise to a portfolio decision. The law of one price is implicit in the two cash-in-advance constraint assumption because, by definition, vendors only accept one currency and therefore only charge one price. As will be in Section 2.4, the exchange rate here serves as a price that clears the currency exchange market rather than a mechanism for enforcing the law of one price. Furthermore, the currency portfolio decision is an interesting one that has not received much attention.[15] However, both the single CIA constraint with the law of one price method and the dual CIA constraints with currency exchange market clearing method deliver the same results for optimal monetary policy.

Using the cash-in-advance constraints (18) and (19), the money laws of motion (6) and (7), and the expressions for the non-proportional transfer in terms of the Home money growth rate (4), country-specific aggregate consumptions can be expressed in the following way:

$$c_{t+1}^h = \frac{m_t^h + (x-1)xM_{t-1}}{P_{t+1}^h} \tag{25}$$

$$c_{t+1}^f = \frac{m_t^f}{P_{t+1}^f} \tag{26}$$

The expression for Home aggregate total consumption is then:

$$c_{t+1} = \left(\frac{m_t^h + (x-1)xM_{t-1}}{P_{t+1}^h}\right)^{1-\theta_h} \left(\frac{m_t^f}{P_{t+1}^f}\right)^{\theta_h} \tag{27}$$

Using the portfolio constraint in (17) to substitute out either m_t^h or m_t^f and substituting in the expression for labor demand from (13), the maximization problem then becomes

$$\max_{m_t^f, w_t(z)} \frac{\left[\left(\frac{w_t(z)^{1-\varepsilon}}{P_{t+1}^h \left(P_t^h\right)^{-\varepsilon}} y_t - \frac{e_t m_t^f - (x-1)xM_{t-1}}{P_{t+1}^h}\right)^{1-\theta_h} \left(\frac{m_t^f}{P_{t+1}^f}\right)^{\theta_h}\right]^{1-\sigma} - 1}{1-\sigma} \dots \tag{28}$$
$$-\chi \left[\left(\frac{w_t(z)}{P_t^h}\right)^{-\varepsilon} y_t\right]^{\xi}$$

[15] Good examples of the currency portfolio choice literature are Engel and Matsumoto (2006) and Evans and Lyons (2005).

The first order conditions with respect to m_t^f and $w_t(z)$, respectively, are:

$$\frac{P_{t+1}^h c_{t+1}^h}{e_t P_{t+1}^f c_{t+1}^f} = \frac{1-\theta_h}{\theta_h} \quad \forall t, z \tag{29}$$

$$(1-\theta_h)\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{w_t(z)}{P_{t+1}^h}\left(c_{t+1}^h\right)^{(1-\theta_h)(1-\sigma)-1}\left(c_{t+1}^f\right)^{\theta_h(1-\sigma)} = \chi\xi\left(n_t(z)\right)^{\xi-1} \quad \forall t, z \tag{30}$$

where equation (29) equates the marginal cost of giving up a Home-currency unit of Home consumption for the marginal benefit of a Home-currency unit of Foreign consumption. Equation (30) equates the real wage with the marginal rate of substitution between consumption and leisure. Because each agent within a country is identical, other than for a differentiated production good, the resulting individual equilibrium wage $w_t(z)$ and the amount of total revenues held in Foreign currency m_t^f will be symmetric across individuals in a given country.

Notice that the first order condition for m_t^f in (29) is implied by the consumption demand functions in (22) and (23) from the expenditure minimization problem This is because the optimal choice of m_t^f in period t is equivalent to the optimal choice of C_{t+1}^h and C_{t+1}^f in period $t+1$. These two decisions are equivalent and take the labor or pricing decision as given.

2.4 Market clearing conditions

This economy has three markets that must clear—the goods market, the money market, and the currency exchange market. The labor market clears by assumption because each differentiated labor supplier is essentially a monopolist who chooses a wage to charge $w_t(z)$ in order to provide labor at the point on the labor demand curve that gives the highest utility. The following paragraphs describe each market and the respective market clearing conditions.

Goods Market. Both Home and Foreign consumers demand goods from both countries. Aggregate supply in the Home and Foreign countries is simply the production of the representative firm y_t and y_t^*, respectively. Define d_t and d_t^* as the world demand for Home-produced goods and Foreign-produced goods, respectively. Goods market clearing requires that aggregate production in each country equal the world demand for that country's goods.

$$y_t = d_t = c_t^h + c_t^{h*} \quad \forall\, t \tag{31}$$

$$y_t^* = d_t^* = c_t^f + c_t^{f*} \quad \forall\, t \tag{32}$$

Money Market. Money market clearing simply requires that money supply equal money demand at the time that goods are purchased.

$$M_t = m_t^h + m_t^{h*} \quad \forall\, t \tag{33}$$

$$M_t^* = m_t^f + m_t^{f*} \quad \forall\, t \tag{34}$$

where M_t and M_t^* are the Home and Foreign aggregate money supplies, respectively, at time t.

Currency Exchange Market. After trade has taken place in the goods market, period-t laborers go to the currency market and make a portfolio decision of how much of each currency to hold. The exchange rate e_t is the price that equates the amount of Foreign currency purchased with Home currency by Home laborers with the amount of Home currency purchased by Foreign laborers with Foreign currency.

$$e_t m_t^f = m_t^{h*} \quad \forall t \tag{35}$$

It is important to note that the exchange rate here is not pinned down by the assumption of the law of one price as in models with a single cash-in-advance constraint, such as Corsetti and Pesenti (2001) and Arseneau (2007). Here, the exchange rate is a price that clears the currency exchange market in period-t. Because of the two cash-in-advance constraints, the law of one price holds by definition. Using the cash-in-advance constraint (19) and its Foreign country analogue, it can be shown that exchange rate market clearing implies that the nominal value of imports equals the nominal value of exports.

$$e_t P^f_{t+1} C^f_{t+1} = P^h_{t+1} C^{h*}_{t+1} \quad \forall t \tag{36}$$

2.5 Equilibrium

This perfect foresight overlapping generations model has one unique nonautarkic steady state equilibrium. As noted in Section 2.1, I avoid discretionary monetary policy in this paper due to the resulting characteristic of multiple equilibria, most of which are unstable sunspot equilibria characterized by expectations traps.[16] Table 1 shows the conditions that must hold in a perfect foresight equilibrium. I define the steady state international equilibrium given both Home and Foreign monetary policy (x, x^*) as follows:

Definition 1 (**Steady State International Equilibrium given** x **and** x^*)**.** A steady state international equilibrium, given Home and Foreign monetary policy (x, x^*) is the set of Home consumption of both Home and Foreign aggregate goods c^h and c^f, Home production y, Home labor n Home portfolio holdings of both Home and Foreign currency m^h and m^f (or rather, as a percentage of initial Home holdings, ϕ and $1-\phi$), the Foreign counterparts $(c^{h*}, c^{f*}, y^*, n^*, m^{h*}, m^{f*})$, Home and Foreign prices and wages $(P^h, P^f, w(z), w(z^*))$, and exchange rate e_t such that:

- **Individual optimization**: Home and Foreign agents choose the wage rate for their differentiated labor input as well as their currency portfolio holdings in order to maximize lifetime utility in (24) and its Foreign counterpart subject to a budget constraint (17) and two cash-in-advance constraints (18) and (19).
- **Firm profit maximization**: Home and Foreign firms choose how much to produce and how much of each type of labor to use given contracted wages and a market price of their good in order to maximize profits according to (12). Wages are contracted in imperfectly competitive labor markets, but the price of the firms good is determined in a perfectly competitive environment according to a zero-profit condition.
- **Market Clearing** The goods markets (31) and (32), money markets (33) and (34), and currency exchange market (35) all clear.

[16]King and Wolman (2004) is a good current reference on multiple equilibria in models of discretionary monetary policy, which builds on the seminal work of Kydland and Prescott (1977) and Barro and Gordon (1983). See also Chatterjee, Cooper, and Ravikumar (1993).

Table 1: Equilibrium conditions given x and x^*

	Home country	Foreign country
(29)	$\frac{P_{t+1}^h c_{t+1}^h}{e_t P_{t+1}^f c_{t+1}^f} = \frac{1-\theta_h}{\theta_h}$	$\frac{e_t P_{t+1}^f c_{t+1}^{f*}}{P_{t+1}^h c_{t+1}^{h*}} = \frac{1-\theta_f}{\theta_f}$
(30)	$(1-\theta_h)\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{w_t(z)}{P_{t+1}^h}\frac{\left(c_{t+1}^h\right)^{(1-\theta_h)(1-\sigma)-1}}{\left(c_{t+1}^f\right)^{-\theta_h(1-\sigma)}} = \chi\xi\left(n_t(z)\right)^{\xi-1}$	$(1-\theta_f)\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{w_t(z^*)}{P_{t+1}^f}\frac{\left(c_{t+1}^{f*}\right)^{(1-\theta_f)(1-\sigma)-1}}{\left(c_{t+1}^{h*}\right)^{-\theta_f(1-\sigma)}} = \chi\xi\left(n_t(z^*)\right)^{\xi-1}$
(13)	$n_t(z) = \left(\frac{w_t(z)}{P_t^h}\right)^{-\varepsilon} y_t$	$n_t(z^*) = \left(\frac{w_t(z^*)}{P_t^f}\right)^{-\varepsilon} y_t^*$
(17)	$w_t(z)n_t(z) = m_t^h + e_t m_t^f$	$w_t(z^*)n_t(z^*) = m_t^{f*} + \frac{m_t^{h*}}{e_t}$
(25)	$c_{t+1}^h = \frac{m_t^h + (x-1)xM_{t-1}}{P_{t+1}^h}$	$c_{t+1}^{f*} = \frac{m_t^{f*} + (x^*-1)x^* M_{t-1}^*}{P_{t+1}^f}$
(26)	$c_{t+1}^f = \frac{m_t^f}{P_{t+1}^f}$	$c_{t+1}^{h*} = \frac{m_t^{h*}}{P_{t+1}^h}$
(21)	$c_{t+1} = \left(c_{t+1}^h\right)^{1-\theta_h}\left(c_{t+1}^f\right)^{\theta_h}$	$c_{t+1}^* = \left(c_{t+1}^{f*}\right)^{1-\theta_f}\left(c_{t+1}^{h*}\right)^{\theta_f}$
	Market clearing conditions	
(31)	$y_t = c_t^h + c_t^{h*}$	
(32)	$y_t^* = c_t^f + c_t^{f*}$	
(33)	$M_t = m_t^h + m_t^{h*}$	
(34)	$M_t^* = m_t^f + m_t^{f*}$	
(35)	$e_t m_t^f = m_t^{h*}$	

Following Cooper and Kempf (2003), let ϕ_t represent the share of wage revenues $w_t(z)n_t(z)$ kept in the form of Home currency in period t, and let $1-\phi_t$ be the share of revenues exchanged for Foreign currency as characterized in the portfolio budget constraint (17). Then the following expressions hold.

$$w_t(z)n_t(z) - m_t^h = e_t m_t^f = (1-\phi_t)\, M_t \tag{37}$$

$$w_t(z^*)n_t(z^*) - m_t^{f*} = \frac{m_t^{h*}}{e_t} = (1-\phi_t^*)\, M_t^* \tag{38}$$

$$m_t^h + \tau_{t+1} = (\phi_t + x - 1)\, M_t \tag{39}$$

$$m_t^{f*} + \tau_{t+1}^* = (\phi_t^* + x^* - 1)\, M_t^* \tag{40}$$

Plugging (37), (38), (39), and (40) into the first order condition (29) and its Foreign country analogue, the unique nonautarkic steady state equilibrium share of currency from sales held for own-country consumption is given by:

$$\phi = 1 - \theta_h x \quad \forall x \in \left(0, \frac{1}{\theta_h}\right) \tag{41}$$

$$1 - \phi = \theta_h x \quad \forall x \in \left(0, \frac{1}{\theta_h}\right) \tag{42}$$

$$\phi^* = 1 - \theta_f x^* \quad \forall x^* \in \left(0, \frac{1}{\theta_f}\right) \tag{43}$$

$$1 - \phi^* = \theta_f x^* \quad \forall x^* \in \left(0, \frac{1}{\theta_f}\right) \tag{44}$$

From the aggregate money laws of motion in (2) and (3) and from the money market clearing conditions in (33) and (34), it is clear that the non-autarkic steady state equilibrium country-specific consumption inflation rates are:

$$\frac{P^h_{t+1}}{P^h_t} = x \tag{45}$$

$$\frac{P^f_{t+1}}{P^f_t} = x^* \tag{46}$$

Furthermore, using the definition of the Home country CPI level P_{t+1} from (15) and its Foreign country analogue, the expressions for the share Home country revenues traded for Foreign currency balances (42) and the share of Foreign country revenues traded for Home currency balances (44), and the currency exchange market clearing condition (35), the Home country CPI growth rate and the Foreign country CPI growth rates can be shown to be equal to their respective countries' money growth rates.[17]

$$\frac{P_{t+1}}{P_t} = x \tag{47}$$

$$\frac{P^*_{t+1}}{P^*_t} = x^* \tag{48}$$

Using (41), (42), (43), and (44), as well as the equilibrium inflation rates from (45) and (46), equilibrium consumption can be derived in terms of steady state em-

[17]See Derivation 3 in Technical Appendix T-1 (available upon request).

ployment from the cash-in-advance constraints as:

$$c^h = (1-\theta_h)n \tag{49}$$

$$c^f = \theta_f n^* \tag{50}$$

$$c^{f*} = (1-\theta_f)n^* \tag{51}$$

$$c^{h*} = \theta_h n \tag{52}$$

where the steady state employment levels n and n^* are characterized below in equations (55) and (56).

The expressions for the steady state international equilibrium employment are then found by solving the two equilibrium forms of the Home first order condition (30) and its Foreign analogue.

$$(1-\theta_h)\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{1}{x}\left[(1-\theta_h)n\right]^{(1-\theta_h)(1-\sigma)-1}\left[\theta_f n^*\right]^{\theta_h(1-\sigma)} = \chi\xi(n)^{\xi-1} \tag{53}$$

$$(1-\theta_f)\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{1}{x^*}\left[(1-\theta_f)n^*\right]^{(1-\theta_f)(1-\sigma)-1}\left[\theta_h n\right]^{\theta_f(1-\sigma)} = \chi\xi(n^*)^{\xi-1} \tag{54}$$

Solving (54) for n^* and plugging it into (53), and doing the symmetric operation for the Foreign country gives the expressions for Home and Foreign equilibrium labor supply:

$$n\left(x,x^*\right) = \Omega_H\left(x\right)^{\frac{\Delta_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}\left(x^*\right)^{\frac{-\Sigma_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{55}$$

$$n^*\left(x^*,x\right) = \Omega_F\left(x^*\right)^{\frac{\Delta_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}\left(x\right)^{\frac{-\Sigma_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{56}$$

where the symbols in (55) and (56) summarize the parameters of the model in the

Table 2: Properties of representative parameters

Symbol	Sign	$\frac{\partial(\cdot)}{\partial\theta_h}$	$\frac{\partial(\cdot)}{\partial\theta_f}$
Δ_h	$(-)$ always	$(+)$ when $\sigma > 1$	
Δ_f	$(-)$ always		$(+)$ when $\sigma > 1$
Σ_h	$(-)$ when $\sigma > 1$ and $\theta_h > 0$	$(-)$ when $\sigma > 1$	
Σ_f	$(-)$ when $\sigma > 1$ and $\theta_f > 0$		$(-)$ when $\sigma > 1$
Ω_h	$(+)$ when $\theta_f > 0$	$(-)$ when $\sigma > 1$ and $\theta_f > 0$	$(+)$ when $\sigma > 1$ and $\theta_h > 0$
Ω_f	$(+)$ when $\theta_h > 0$	$(+)$ when $\sigma > 1$ and $\theta_f > 0$	$(-)$ when $\sigma > 1$ and $\theta_h > 0$
$\Delta_h\Delta_f - \Sigma_h\Sigma_f$	$(+)$ always	$(-)$ when $\sigma > 1$	$(-)$ when $\sigma > 1$

Note: The results from this table are derived in Derivation 4 in Technical Appendix T-1 and are available upon request.

following way:

$$\Delta_h = (1-\theta_h)(1-\sigma) - \xi \tag{57}$$

$$\Delta_f = (1-\theta_f)(1-\sigma) - \xi \tag{58}$$

$$\Sigma_h = \theta_h(1-\sigma) \tag{59}$$

$$\Sigma_f = \theta_f(1-\sigma) \tag{60}$$

$$\Omega_h = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\chi\xi}{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}(\theta_f)^{\theta_h(1-\sigma)}} \tag{61}$$

$$\Omega_f = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\chi\xi}{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}(\theta_h)^{\theta_f(1-\sigma)}} \tag{62}$$

$$\Omega_H = (\Omega_h)^{\frac{\Delta_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}(\Omega_f)^{\frac{-\Sigma_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{63}$$

$$\Omega_F = (\Omega_f)^{\frac{\Delta_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}(\Omega_h)^{\frac{-\Sigma_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{64}$$

The signs of these expressions and their derivatives with respect to the openness parameters θ_h and θ_f are given in Table 2. From the signs of the representative parameters, it is clear that steady state equilibrium Home employment n decreases in x always and increases in x^* when $\sigma > 1$.

Looking at the equation for Home labor supply in (55), the sign of Σ_h determines how Foreign monetary policy affects the real economy in the Home country.

$$\Sigma_h = \begin{cases} > 0 & \text{if} \quad \theta_h \in (0, 0.5] \quad \text{and} \quad \sigma \in (0,1) \\ = 0 & \text{if} \quad \theta_h = 0 \quad \text{or} \quad \sigma = 1 \\ < 0 & \text{if} \quad \theta \in (0, 0.5] \quad \text{and} \quad \sigma > 1 \end{cases} \tag{65}$$

The third case is the most common in which $\Sigma_h < 0$, implying that Foreign inflation causes an increase in the equilibrium level of Home production and, therefore, an increase in equilibrium consumption of the Home good by both Home and Foreign consumers.

If one were to make the strong assumption that the coefficient of relative risk aversion σ were less than one, the first case in (65) occurs in which Foreign inflation causes a decrease in the equilibrium level of Home production. Lastly, it is interesting to notice the cases in which Foreign monetary policy has no real effect on the Home country ($\Sigma = 0$). Obviously, when the economies do not trade with each other, $\theta_h = 0$, Foreign monetary policy will be neutral. But it is interesting to note that the case of log utility ($\sigma = 1$) also induces the real neutrality of Foreign monetary policy.

The monetary authority in each country seeks to maximize the lifetime utility of a representative agent in this economy by choosing Home monetary policy x given Foreign monetary policy x^*. Define $V(x, x^*)$ as the lifetime utility of a representative agent. The objective of the Home monetary authority is then:

$$\max_x \; V(x, x^*) = \max_x \; \frac{\left([(1-\theta_h) n(x, x^*)]^{1-\theta_h} [\theta_f n^*(x^*, x)]^{\theta_h} \right)^{1-\sigma} - 1}{1-\sigma} - \chi n(x, x^*)^{\xi} \tag{66}$$

Definition 2 (**Home Country Steady State Monetary Equilibrium**)**.** A Home country steady state monetary equilibrium is a function for the optimal Home money growth rate $\hat{x}(x^*)$ given the Foreign money growth rate such that:

- the individual steady state equilibrium conditions from Definition 1 hold for each country,
- the Home monetary authority chooses x to maximize the lifetime utility of the

representative agent of its country as in equation (66).

Definition 2 can be thought of as a monetary partial equilibrium in a world monetary environment because it implies a best response function for Home monetary policy that is a function of any level of Foreign monetary policy. Taking the derivative of (66), the resulting solution for optimal Home monetary policy is:[18]

$$\begin{aligned}\hat{x} &= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{\Delta_f}{(1-\theta_h)\Delta_f-\theta_h\Sigma_f}\\ &= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{(1-\theta_f)(1-\sigma)-\xi}{(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_h)\xi}\end{aligned} \tag{67}$$

The analogous solution for the Foreign monetary authority is:

$$\begin{aligned}\hat{x}^* &= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{\Delta_h}{(1-\theta_f)\Delta_h-\theta_f\Sigma_h}\\ &= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{(1-\theta_h)(1-\sigma)-\xi}{(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_f)\xi}\end{aligned} \tag{68}$$

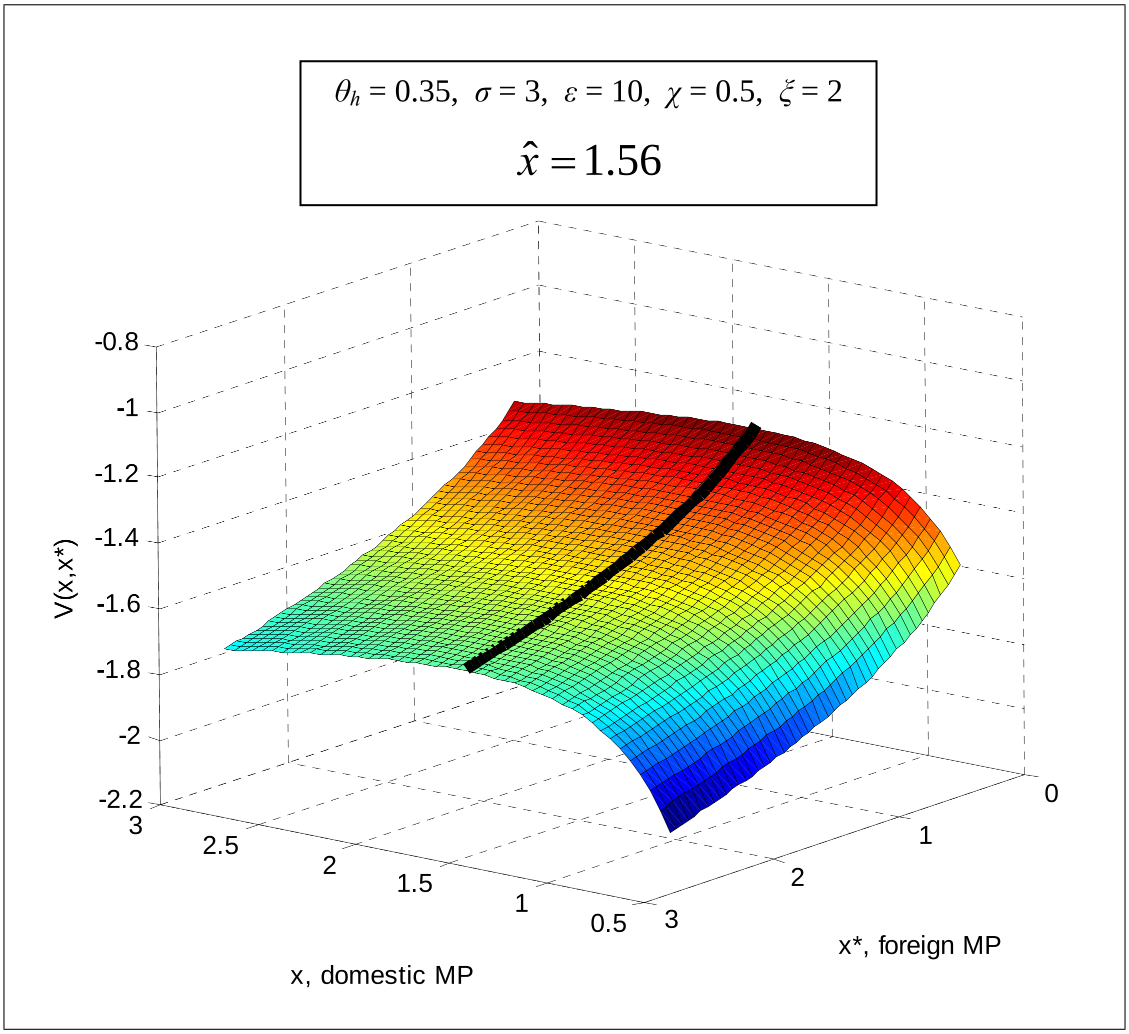
The first characteristic to note about the optimal Home monetary policy function in (67) is that it is independent of Foreign monetary policy x^*. That is, the optimal level of the Home money growth rate does not change with changes in the Foreign money growth rate and is a dominant strategy equilibrium.[19]

This dominant strategy equilibrium is shown in Figure 2 which plots the lifetime utility of a representative Home agent from (66) as a function of Home inflation x and Foreign inflation x^*. The parameters $(\theta, \sigma, \varepsilon, \chi, \xi)$ are simply chosen to reflect values estimated in the empirical literature in order to make a simple example. The dark line running across the top of Figure 2 represents the Home monetary policy best response function from (67). The optimal Home inflation level at the selected parameter values is a constant $\hat{x} = 1.56$, which is not overly high given that the duration of a period is a generation. Because each country's best response function for monetary policy is a dominant strategy equilibrium, the world Nash monetary

[18]See Derivation 5 in Technical Appendix T-1 (available upon request).

[19]Derivation 5 in Technical Appendix T-1 details why $\hat{x}$ is independent of x^* (available upon request).

Figure 2: Home lifetime utility V as a function of x and x^*



equilibrium is the same as the country partial monetary equilibrium.

The main question of this paper is whether openness is inflationary. The following proposition answers this question with regard to both absolute inflation rate (Home country consumer price growth rate) and the real exchange rate inflation (Home country consumer price growth rate over Foreign country consumer price growth rate).

Proposition 1 (**Monetary response to changes in openness**)**.** The equilibrium optimal Home money growth rate $\hat{x}$ in (67) increases with more Home openness in the form of a higher level of θ_h and in response to more Foreign openness in the form of a higher level of θ_f. The argument for the Foreign country is symmetric. However, when θ_h increases, the increase in $\hat{x}$ is greater than the increase in $\hat{x}^*$. Conversely, when θ_f increases, the increase in $\hat{x}^*$ is greater than the increase in $\hat{x}$.

Proof. See Appendix A-1. □

Because the Home country CPI growth rate (P_{t+1}/P_t) is equal to the Home money growth rate x, an increase in θ_h increases Home country inflation as well as Foreign country inflation. From the perspective of the Home monetary authority, if the Home marginal utility of Home consumption decreases relative to the Home marginal utility of Foreign consumption as is the case when θ_h increases while θ_f remains constant (see first order condition (30)), Home country agents bear a smaller proportion of the inflation tax. In effect, higher θ_h increases the welfare benefits from higher money growth rates to the Home country and lowers the costs. Consequently, the optimal response by the Home monetary authority is to raise the Home money growth rate or the CPI inflation rate in response to a higher degree of openness.

The next two propositions further explain how the level of imperfect competition in a country's labor market, as parameterized by the elasticity of substitution among the differentiated labor inputs ε, influences the optimal money growth rate x and the real outcomes of the economy in equilibrium.

Proposition 2 (Deflationary bias of imperfect competition). Both the optimal Home money growth rate $\hat{x}$ and the optimal Foreign money growth rate $\hat{x}^*$ decrease as the level of imperfect competition increases (as ε decreases). Furthermore, there exist two critical within-country elasticities of substitution for the Home country and Foreign country $(\bar{\varepsilon}, \bar{\varepsilon}^*)$ such that $\hat{x} = 1$ when $\varepsilon = \bar{\varepsilon}$ and $\hat{x}^* = 1$ when $\varepsilon = \bar{\varepsilon}^*$. That is, these two critical levels of the imperfect competition parameter implement the Friedman Rule in the Home and Foreign country, respectively.

$$\bar{\varepsilon} = \frac{(1-\theta_f)(1-\sigma)-\xi}{\theta_h(1-\sigma-\xi)} \tag{69}$$

$$\bar{\varepsilon}^* = \frac{(1-\theta_h)(1-\sigma)-\xi}{\theta_f(1-\sigma-\xi)} \tag{70}$$

Proof. See Appendix A-1. □

This result that the level imperfect competition induces a deflationary bias in monetary policy has been shown recently in a different model by Arseneau (2007).

Lastly, Proposition 3 highlights the relationship between the level of market power held by producers within a country and the monopoly power held by the each monetary authority in international markets.

Proposition 3 (Market power neutrality). In the case of symmetric countries $\theta_h = \theta_f$, the steady state equilibrium levels of employment n and n^* are not affected by the level of imperfect competition ε within both countries.

Proof. See Appendix A-1. □

Proposition 3 says that the real outcomes in each country $(n, n^*, c^h, c^f, c^{h*}, c^{f*})$ are the same regardless of whether the countries are characterized by perfect competition $\varepsilon = \infty$ or whether any degree of monopoly power is enjoyed by labor suppliers $\varepsilon < \infty$. The implication of this result is that if any monopoly rents available to Home or Foreign agents are not collected through producer price setting, the remainder will be collected by the monetary authority raising prices. As stated in Proposition 2, a level of imperfect competition exists at which all the monopoly rents are collected through producer price setting alone. That is, inflation generated by the monetary authority increasing the money growth rate is not needed.

3 Empirical Test

The inflation data used here come from the IMF's International Financial Statistics. I use the GDP deflator whenever available, and I use the CPI otherwise. A proxy for openness is an issue is subject to more debate. This paper follows the convention that has been used in much of the previous literature, which is to treat the import share (imports as a share of GDP) as the proxy for a country's level of openness.[20] The results in this section are not very sensitive to whether exports are included in some way in the numerator of this measure.

Figures 3 and 4 show the median inflation rate and median import share for the 30 OECD countries, as well as the 25-percent to 75-percent quartile range, over the

[20] This is the convention used in Romer (1993) and Lane (1997).

period from 1960 to 2005. Notice two facts from the inflation picture in Figure 3. First, the median inflation rate has been declining among OECD countries since the early 1970s. In particular, the decade from the early 1970s to the early 1980s can be classified as a high-inflation period. The second fact is that the variance in inflation rates across countries has been declining as well since the early 1970s.

Figure 3: Median inflation rate for 30 OECD countries

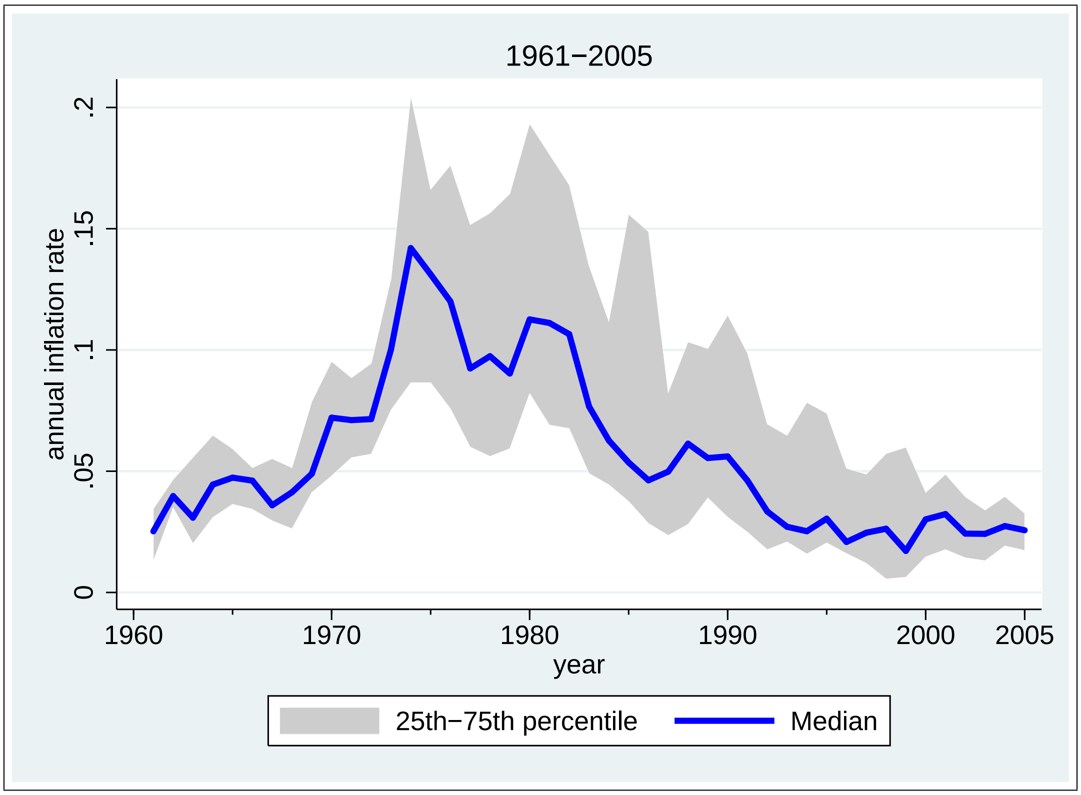
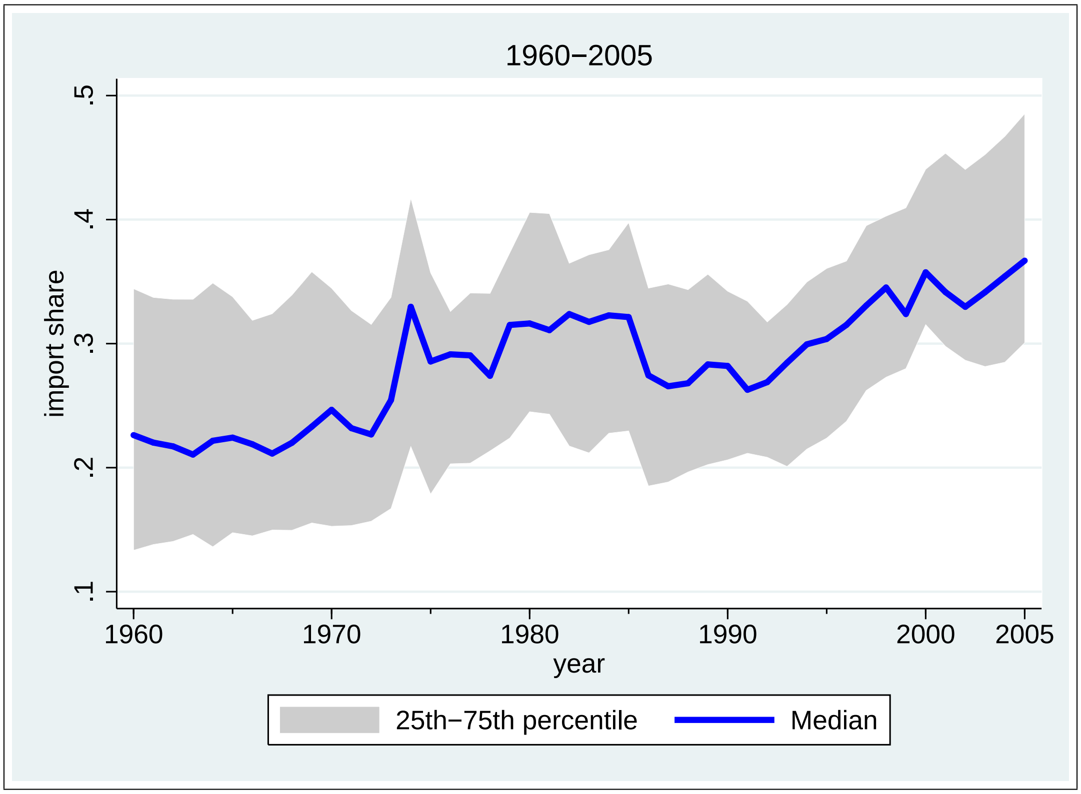


Figure 4 shows that the median import share for the 30 OECD countries has been rising since the early 1970s, but that the variance in the share has stayed relatively constant. Taken together, these two figures reflect a negative correlation across all the countries between openness and inflation. However, Figure 5 shows that this relationship is not obvious even with respect to simple correlation if one looks at a cross section of countries. Neither Figure 3 nor Figure 4 changes much if more countries are included.

Figure 5 shows the average annual inflation rate and the average annual import share for each of the 30 OECD countries averaged over the 1982 to 2005 period. The

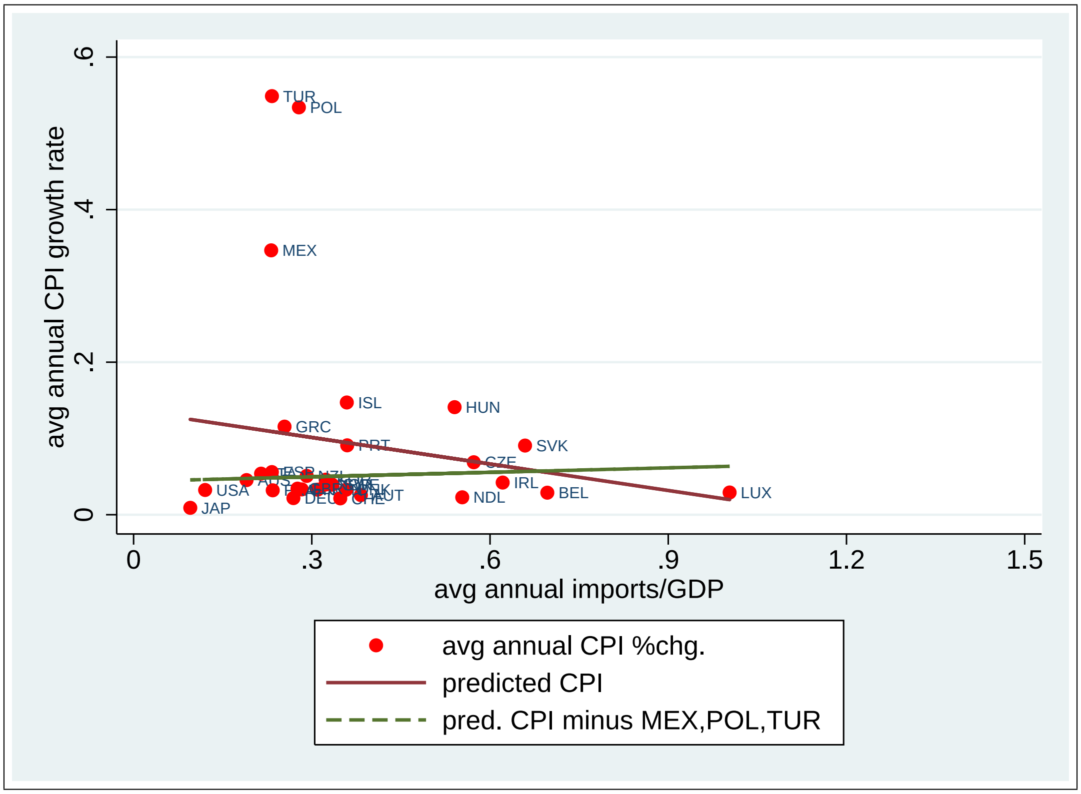
Figure 4: Median import share for 30 OECD countries



solid downward sloping line is the linear fitted line for all 30 countries, reflecting a negative correlation between openness and inflation. However, when the high-inflation outliers of Mexico, Poland, and Turkey are excluded, the relationship becomes positive as evidenced by the upward sloping dashed line. Both relationships get closer to zero as the sample years become smaller and smaller. The point to take from Figure 5 is that the relationship between openness and inflation is not obvious, even when looking only at the simple correlation level.

My first empirical test is to run Romer's (1993) regressions with the addition of controlling for the level of imperfect competition. The imperfect competition data come from two sources. First, I use estimates of both manufacturing and nonmanufacturing markups from Høj, Jimenez, Maher, Nicoletti, and Wise (2007) for 17 OECD countries. Obviously, this presents a problem because of the small sample size. The other imperfect competition data come from Blanchflower (2006), Visser (2006), for Economic Cooperation and Development (2004), and Organisation (1998).

Figure 5: Import share vs. CPI for 30 OECD countries: annual avg. for 1982 to 2005



The first column of Table 3 shows Romer's (1993) baseline specification with the same data and over the same sample period. The subsequent columns of Table 3 show the Romer regression augmented by the various controls for the level of imperfect competition. Using this sample period, the negative relationship between openness and inflation is robust to any specification I try.

However, Table 5 presents a different story. This table runs the Romer regression in a different sample period, 1988 to 2002. As was discussed in Figure 3, the period from Romer (1993) was characterized by abnormally high inflation and included the rejection of the pegged exchange rate system. Using the more current sample period, the negative relationship between openness and inflation disappears. Descriptive statistics for the variables in these regressions are reported in Table 4.

In each of the specifications in which I control for the level of imperfect competition, the sign of the coefficient on openness is positive. This result must be taken

Table 3: Regression of openness on inflation: 1973 to 1987

	log average inflation 1973-1987			
	Romer (1993)	Union Mem. Rt.	(1) Union Cov. Rt.	(2) Union Cov. Rt.
Import share	-1.130***	-1.719***	-1.263*	-1.828**
(1973-1987)	(0.272)	(0.393)	(0.715)	(0.703)
Real per cpta. inc.	0.038	0.051	-0.912***	-0.037
(1980)	(0.070)	(0.162)	(0.218)	(0.187)
OECD dummy	-0.446**	-0.857***		-1.205**
	(0.188)	(0.270)		(0.453)
Union mem. rate		0.013***		
(1978-1988)		(0.005)		
Union cov. rate			0.006	
(1980-1990)			(0.004)	
Union cov. rate				0.011**
(1980-1995)				(0.006)
Countries	114	64	20	44
R-squared	0.167	0.335	0.532	0.352

with some caution because the Romer regression with the sample restricted to the corresponding imperfect competition sample also has a positive coefficient. So it is not necessarily controlling for imperfect competition that makes the relationship positive. The sample of countries declines appreciably with each different imperfect competition measure.

Also, none of the coefficients on openness in the more current sample period is statistically significant. These regressions must be taken as evidence that a positive

Table 4: Descriptive statistics of regression variables

Variable	Mean	Std. Dev.	Min.	Max.	N
avg. inflation (88-02)	0.158	0.229	0.004	1.360	124
avg. import share (88-02)	0.429	0.225	0.087	1.314	124
Real per cpta. inc. (95)	7,947	7,623	531	33,757	124
Union mem. rate (88-02)	24.0	17.0	2	80.9	84
Union cov. rate (90-00)	45.1	29.4	1.5	95.0	45
Mfct. markup (04)	24.4	5.6	16.0	38.0	16
Nonmfct. markup (04)	12.4	2.6	7.0	18.0	17

Table 5: **Regression of openness on inflation: 1988 to 2002**

	log average inflation 1988-2002				
	Romer updated	Union Mem. Rt.	Union Cov. Rt.	Mfct. Markup	Nonmfct. Markup
Import share (1988-2002)	-0.343 (0.423)	0.088 (0.512)	0.062 (0.549)	1.084 (0.680)	1.317* (0.666)
Real per cpta. inc. (1995)	-0.340*** (0.085)	-0.526*** (0.127)	-0.805*** (0.129)	-1.384 (0.873)	-1.293 (0.769)
Union mem. rate (1988-2002)		0.008 (0.008)			
Union cov. rate (1990-2000)			0.004 (0.005)		
Mfct. markup (2004)				0.004 (0.028)	
Nonmfct. markup (2004)					0.066 (0.060)
Countries	124	84	45	16	17
R-squared	0.134	0.191	0.546	0.248	0.340

relationship between openness and inflation may exist. Furthermore, additional data on imperfect competition would be valuable for this question as well as many others. And the additional data does not have to be for as broad a group of countries if some degree of panel dimension can be obtained.

4 Conclusion

This paper proposes a model that predicts that increased openness to trade in a country will increase its equilibrium inflation rate. Supporting evidence for this theoretical result is then provided by some empirical tests using more current data than the previous studies. These results run contrary to the negative relationship between openness and inflation proposed by theoretical and empirical work in the broadest vein of this literature.

This work begs the question of what is the optimal degree of openness for a country. This is a conceptually simple exercise to perform in this framework, although it is analytically quite involved. The parameter for openness in this paper θ incorporates both individual preferences and policy choices such as barriers to trade. But θ could be endogenized as fiscal decision that takes place before the monetary authority chooses the money growth rate.

APPENDIX

A-1 Proofs

Proof that labor supply always equals labor demand (this one not finished). The characteristic of individual labor supply always equaling labor supply simplifies the individual's lifetime utility optimization problem. Because of the Dixit-Stiglitz differentiated labor structure, each worker is a monopolist who faces a demand curve that incorporates the prices of substitutes for the differentiated labor. In the firm demand equation (13) is:

$$n_t(z) = \left(\frac{w_t(z)}{P_t^h}\right)^{-\varepsilon} y_t \quad \forall\, t, z$$

It is clear that demand for specific z-type differentiated labor $n_t(z)$ is a strictly decreasing function of the contracted wage $w_t(z)$ given that the elasticity of substitution $\varepsilon \geq 1$. The expression for the price charged by the firm for the good from (14) shows that P_t^h is an increasing function of each type of differentiated labor. However, because the contracted wage for each individual type of labor has a measure-zero effect on the price, price is independent of changes in any one wage. [I think the best way to prove supply equals demand is proof by contradiction. Use general utility function.] □

Proof of Proposition 1: Monetary response to changes in openness. Taking the derivative of the expression for $\hat{x}$ in (67) with respect to θ_h and θ_f gives the following results:

$$\hat{x} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\Delta_f}{(1-\theta_h)\Delta_f - \theta_h \Sigma_f} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{(1-\theta_f)(1-\sigma) - \xi}{(1-\theta_h-\theta_f)(1-\sigma) - (1-\theta_h)\xi}$$

$$\frac{\partial \hat{x}}{\partial \theta_h} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\Delta_f (1-\sigma-\xi)}{\left[(1-\theta_h)\Delta_f - \theta_h \Sigma_f\right]^2} > 0$$

$$\frac{\partial \hat{x}}{\partial \theta_f} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\theta_h (1-\sigma)(1-\sigma-\xi)}{\left[(1-\theta_h)\Delta_f - \theta_h \Sigma_f\right]^2} > 0$$

Taking the derivative of the expression for $\hat{x}^*$ in (68) with respect to θ_f and θ_h gives the following results:

$$\hat{x}^* = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\Delta_h}{(1-\theta_f)\Delta_h - \theta_f \Sigma_h} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{(1-\theta_h)(1-\sigma) - \xi}{(1-\theta_h-\theta_f)(1-\sigma) - (1-\theta_f)\xi}$$

$$\frac{\partial \hat{x}^*}{\partial \theta_f} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\Delta_h (1-\sigma-\xi)}{\left[(1-\theta_f)\Delta_h - \theta_f \Sigma_h\right]^2} > 0$$

$$\frac{\partial \hat{x}^*}{\partial \theta_h} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\theta_f (1-\sigma)(1-\sigma-\xi)}{\left[(1-\theta_f)\Delta_h - \theta_f \Sigma_h\right]^2} > 0$$

Now the proposition that when θ_h increases, the increase in $\hat{x}$ is greater than the increase in $\hat{x}^*$, simply means that $\frac{\partial\left(\frac{\hat{x}}{\hat{x}^*}\right)}{\partial\theta_h} > 0$.

$$\begin{aligned}
\frac{\partial\left(\frac{\hat{x}}{\hat{x}^*}\right)}{\partial\theta_h} &= \frac{\partial\hat{x}}{\partial\theta_h}\left[\frac{1}{\hat{x}^*}\right] - \frac{\partial\hat{x}^*}{\partial\theta_h}\left[\frac{\hat{x}}{(\hat{x}^*)^2}\right] \\
&= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{\Delta_f(1-\sigma-\xi)}{[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]^2}\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{(1-\theta_f)\Delta_h-\theta_f\Sigma_h}{\Delta_h}\ldots \\
&\quad -\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{\Sigma_f(1-\sigma-\xi)}{[(1-\theta_f)\Delta_h-\theta_f\Sigma_h]^2}\left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{\Delta_f\left[(1-\theta_f)\Delta_h-\theta_f\Sigma_h\right]^2}{\Delta_h^2\left[(1-\theta_h)\Delta_f-\theta_h\Sigma_f\right]}\right) \\
&= \frac{\Delta_f(1-\sigma-\xi)[(1-\theta_f)\Delta_h-\theta_f\Sigma_h]}{\Delta_h[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]^2} - \frac{\Delta_f\Sigma_f(1-\sigma-\xi)}{\Delta_h^2[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]} \\
&= \frac{\Delta_h\Delta_f(1-\sigma-\xi)[(1-\theta_f)\Delta_h-\theta_f\Sigma_h]-\Delta_f\Sigma_f(1-\sigma-\xi)[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]}{\Delta_h^2[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]^2} \\
&= \Delta_f(1-\sigma-\xi)\left(\frac{\Delta_h[(1-\theta_f)\Delta_h-\theta_f\Sigma_h]-\Sigma_f[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]}{\Delta_h^2[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]^2}\right) \\
&= \Delta_f(1-\sigma-\xi)\left(\frac{\Delta_h[(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_f)\xi]-\Sigma_f[(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_h)\xi]}{\Delta_h^2[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]^2}\right) \\
\frac{\partial\left(\frac{\hat{x}}{\hat{x}^*}\right)}{\partial\theta_h} &= \Delta_f(1-\sigma-\xi)\left(\frac{(\Delta_h-\Sigma_f)(1-\theta_h-\theta_f)(1-\sigma)+\xi\left[\Sigma_f(1-\theta_h)-\Delta_h(1-\theta_f)\right]}{\Delta_h^2[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]^2}\right) > 0
\end{aligned}$$

The last line is true because $\Delta_h - \Sigma_f < 0$ and $\Sigma_f(1-\theta_h) - \Delta_h(1-\theta_f) > 0$. □

Proof of Proposition 2: Deflationary bias of imperfect competition. From (67) and (68):

$$\begin{aligned}
\frac{\partial\hat{x}}{\partial\varepsilon} &= \left(\frac{1}{\varepsilon^2}\right)\frac{\Delta_f}{(1-\theta_h)\Delta_f-\theta_h\Sigma_f} > 0 \\
\frac{\partial\hat{x}^*}{\partial\varepsilon} &= \left(\frac{1}{\varepsilon^2}\right)\frac{\Delta_h}{(1-\theta_f)\Delta_h-\theta_f\Sigma_h} > 0
\end{aligned}$$

Then, to find the respective levels of ε that induce the Home and Foreign monetary authorities, respectively, to set their money growth rates equal to 1 is found by solving (67) and (68) for ε when $\hat{x} = 1$ and when $\hat{x}^* = 1$.

$$\begin{aligned}
\bar{\varepsilon}: \quad 1 &= \left(\frac{\bar{\varepsilon}-1}{\bar{\varepsilon}}\right)\frac{(1-\theta_f)(1-\sigma)-\xi}{(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_h)\xi} \\
\bar{\varepsilon}^*: \quad 1 &= \left(\frac{\bar{\varepsilon}^*-1}{\bar{\varepsilon}^*}\right)\frac{(1-\theta_h)(1-\sigma)-\xi}{(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_f)\xi}
\end{aligned}$$

Solving these two equations for $\bar{\varepsilon}$ and $\bar{\varepsilon}^*$, respectively, gives the results in (69) and

(70).

$$\bar{\varepsilon} = \frac{\Delta_f}{\Sigma_h - \theta_h \xi} = \frac{(1-\theta_f)(1-\sigma) - \xi}{\theta_h(1-\sigma-\xi)}$$
$$\bar{\varepsilon}^* = \frac{\Delta_h}{\Sigma_f - \theta_f \xi} = \frac{(1-\theta_h)(1-\sigma) - \xi}{\theta_f(1-\sigma-\xi)}$$

□

Proof of Proposition 3: Market power neutrality. When the Home and Foreign Country are symmetric $\theta_h = \theta_f = \theta$, the equilibrium employment level is given by:

$$n = \left[\frac{\chi\xi}{(1-\theta)^{(1-\theta)(1-\sigma)}\theta^{\theta(1-\sigma)}}\left(\frac{\varepsilon}{\varepsilon-1}\right)\right]^{\frac{\Delta-\Sigma}{\Delta^2-\Sigma^2}} (\hat{x})^{\frac{\Delta}{\Delta^2-\Sigma^2}} (\hat{x}^*)^{\frac{-\Sigma}{\Delta^2-\Sigma^2}}$$

where $\Delta = (1-\theta)(1-\sigma) - \xi$ and $\Sigma = \theta(1-\sigma)$. The expressions for the optimal money growth rates in this symmetric case are given by:

$$\hat{x} = \hat{x}^* = \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{\Delta}{(1-\theta)\Delta - \theta\Sigma}$$

Now the equilibrium employment level can be written as:

$$\begin{aligned} n = n^* &= \left[\frac{\chi\xi}{(1-\theta)^{(1-\theta)(1-\sigma)}\theta^{\theta(1-\sigma)}}\right]^{\frac{1}{1-\sigma-\xi}}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{1}{1-\sigma-\xi}}\left(\frac{\varepsilon-1}{\varepsilon}\right)^{\frac{1}{1-\sigma-\xi}}\left[\frac{\Delta}{(1-\theta)\Delta-\theta\Sigma}\right]^{\frac{1}{1-\sigma-\xi}} \\ &= \left[\left(\frac{\chi\xi}{(1-\theta)^{(1-\theta)(1-\sigma)}\theta^{\theta(1-\sigma)}}\right)\left(\frac{\Delta}{(1-\theta)\Delta-\theta\Sigma}\right)\right]^{\frac{1}{1-\sigma-\xi}} \end{aligned}$$

It is clear that neither n nor n^* is a function of the level of imperfect competition ε. And because the equilibrium consumption levels are simply constant fractions of the output level, consumption is also not affected by changes in the level of imperfect competition. □

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TECHNICAL APPENDIX

T-1 Derivations

Derivation 1 (**Firm Demand for differentiated labor input** z)**.** The demand function for individual differentiated labor input $n_t(z)$ by firms from (13) is derived in the following way. Firms choose the amount of each type of differentiated labor $n_t(z)$ given the contracted wages $w_t(z)$ and the perfect competition selling price of the output P_t^h.

$$
\begin{aligned}
\max_{n_t(z)} \; \pi_t &= P_t^h \left(\int_0^1 n_t(z)^{\frac{\varepsilon-1}{\varepsilon}} \, dz \right)^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 w_t(z) n_t(z) \, dz \quad \forall \, t \\
\frac{\partial \pi_t}{\partial n_t(z)} \quad &\Rightarrow \quad P_t^h \left(\int_0^1 n_t(z)^{\frac{\varepsilon-1}{\varepsilon}} \, dz \right)^{\frac{1}{\varepsilon-1}} n_t(z)^{-\frac{1}{\varepsilon}} - w_t(z) = 0 \quad \forall \, t, z \\
&\Rightarrow \quad n_t(z) = \left(\frac{w_t(z)}{P_t^h} \right)^{-\varepsilon} y_t \quad \forall \, t, z
\end{aligned}
$$

Derivation 2 (**Firm output price** P_t^h)**.** The expression for the price level of the consumption good P_t^h is pinned down by the zero profit condition. Set the profit function equal to zero, substitute in the expression of differentiated input demand (13), and solve for P_t^h.

$$
\begin{aligned}
P_t^h : \quad & \pi_t = 0 \quad \forall t \\
\Rightarrow \quad & P_t^h \left(\int_0^1 \left[\left(\frac{w_t(z)}{P_t^h} \right)^{-\varepsilon} y_t \right]^{\frac{\varepsilon-1}{\varepsilon}} dz \right)^{\frac{\varepsilon}{\varepsilon-1}} = \int_0^1 w_t(z) \left(\frac{w_t(z)}{P_t^h} \right)^{-\varepsilon} y_t \, dz \quad \forall t \\
\Rightarrow \quad & \left(P_t^h \right)^{1+\varepsilon} y_t \left(\int_0^1 w_t(z)^{1-\varepsilon} dz \right)^{\frac{\varepsilon}{\varepsilon-1}} = \left(P_t^h \right)^{\varepsilon} y_t \left(\int_0^1 w_t(z)^{1-\varepsilon} dz \right) \quad \forall t \\
\Rightarrow \quad & P_t^h = \left(\int_0^1 w_t(z)^{1-\varepsilon} dz \right)^{\frac{1}{1-\varepsilon}} \quad \forall t
\end{aligned}
$$

Derivation 3 (**Steady State Equilibrium Home and Foreign CPI growth rates**)**.** The steady state equilibrium Home and Foreign country consumer price growth rates as shown in (47) and (48) are derived in the following way. The Home consumer price level P_t^h is derived in Derivation 2 in this appendix, and takes the following form as in (15):

$$
P_{t+1} = \frac{1}{(1-\theta_h)^{1-\theta_h} \theta_h^{\theta_h}} \left(P_{t+1}^h \right)^{1-\theta_h} \left(e_t P_{t+1}^f \right)^{\theta_h}
$$

Dividing P_{t+1} by P_t gives the following expression for the Home country consumer price growth rate:

$$\begin{aligned}\frac{P_{t+1}}{P_t} &= \left(\frac{P^h_{t+1}}{P^h_t}\right)^{1-\theta_h}\left(\frac{e_t P^f_{t+1}}{e_{t-1}P^f_t}\right)^{\theta_h} \\ &= (x)^{1-\theta_h}\left(\frac{e_t}{e_{t-1}}x^*\right)^{\theta_h}\end{aligned}$$

Using the currency exchange market clearing condition (35) and plugging in the equilibrium expressions for m^f_t and m^{h*}_t, the steady state equilibrium expression for the growth rate of the exchange rate is:

$$\frac{e_t}{e_{t-1}} = \frac{x}{x^*}$$

Thus, the expression for the steady state equilibrium consumer price growth rate in the Home country is:

$$\frac{P_{t+1}}{P_t} = x$$

And by symmetry, the steady state equilibrium consumer price growth rate in the Foreign country is:

$$\frac{P^*_{t+1}}{P^*_t} = x^*$$

It is the steady-state exchange rate growth expression that cancels out the effects of the other country's prices in each consumer price growth rate expression.

Derivation 4 (**Sign of parameter objects and derivatives with respect to θ_h and θ_f**)**.** Here I derive the derivatives of the parameter summary objects with respect to θ_h and θ_f. A review of the objects and their representation is the following:

$$\begin{aligned}\Delta_h &= (1-\theta_h)(1-\sigma)-\xi \\ \Delta_f &= (1-\theta_f)(1-\sigma)-\xi \\ \Sigma_h &= \theta_h(1-\sigma) \\ \Sigma_f &= \theta_f(1-\sigma) \\ \Omega_h &= \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\chi\xi}{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}(\theta_f)^{\theta_h(1-\sigma)}} \\ \Omega_f &= \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\chi\xi}{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}(\theta_h)^{\theta_f(1-\sigma)}} \\ \Omega_H &= (\Omega_h)^{\frac{\Delta_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}\,(\Omega_f)^{\frac{-\Sigma_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \\ \Omega_F &= (\Omega_f)^{\frac{\Delta_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}\,(\Omega_h)^{\frac{-\Sigma_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}\end{aligned}$$

The signs of the representative parameter objects and their derivatives with respect to θ_h and θ_f are the following:

$$\Delta_h < 0 \text{ always}, \quad \frac{\partial \Delta_h}{\partial \theta_h} = -(1-\sigma) > 0 \text{ when } \sigma > 1$$
$$\Delta_f < 0 \text{ always}, \quad \frac{\partial \Delta_f}{\partial \theta_f} = -(1-\sigma) > 0 \text{ when } \sigma > 1$$
$$\Sigma_h < 0 \text{ when } \sigma > 1 \text{ and } \theta_h > 0, \quad \frac{\partial \Sigma_h}{\partial \theta_h} = 1-\sigma < 0 \text{ when } \sigma > 1$$
$$\Sigma_f < 0 \text{ when } \sigma > 1 \text{ and } \theta_f > 0, \quad \frac{\partial \Sigma_f}{\partial \theta_f} = 1-\sigma < 0 \text{ when } \sigma > 1$$
$$\Omega_h > 0 \text{ when } \theta_f > 0,$$
$$\frac{\partial \Omega_h}{\partial \theta_h} = \Omega_h \frac{\partial log(\Omega_h)}{\partial \theta_h} = \Omega_h(1-\sigma)\left[1 + log\left(\frac{1-\theta_h}{\theta_f}\right)\right] < 0 \text{ when } \sigma > 1 \text{ and } \theta_f > 0$$
$$\frac{\partial \Omega_h}{\partial \theta_f} = \Omega_h \frac{\partial log(\Omega_h)}{\partial \theta_f} = -\Omega_h(1-\sigma)\frac{\theta_h}{\theta_f} > 0 \text{ when } \sigma > 1 \text{ and } \theta_h > 0$$
$$\Omega_f > 0 \text{ when } \theta_h > 0,$$
$$\frac{\partial \Omega_f}{\partial \theta_f} = \Omega_f \frac{\partial log(\Omega_f)}{\partial \theta_f} = \Omega_f(1-\sigma)\left[1 + log\left(\frac{1-\theta_f}{\theta_h}\right)\right] < 0 \text{ when } \sigma > 1 \text{ and } \theta_h > 0$$
$$\frac{\partial \Omega_f}{\partial \theta_h} = \Omega_f \frac{\partial log(\Omega_f)}{\partial \theta_h} = -\Omega_f(1-\sigma)\frac{\theta_f}{\theta_h} > 0 \text{ when } \sigma > 1 \text{ and } \theta_f > 0$$
$$\Delta_h\Delta_f - \Sigma_h\Sigma_f = (1-\theta_h-\theta_f)(1-\sigma)^2 - (1-\sigma)\xi(2-\theta_h-\theta_f) + \xi^2 > 0 \text{ always}$$
$$\frac{\partial(\Delta_h\Delta_f - \Sigma_h\Sigma_f)}{\partial \theta_h} = \frac{\partial(\Delta_h\Delta_f - \Sigma_h\Sigma_f)}{\partial \theta_f} = -(1-\sigma)(1-\sigma-\xi) < 0 \text{ when } \sigma > 1$$

Derivation 5 (Optimal monetary rules). The optimal monetary policy rules (67) and (68) are derived by having the monetary authority maximize the equilibrium utility of a representative agent in its own country with respect to its money growth rate. Below is the solution for the problem of the Home monetary authority, but the Foreign monetary authority's problem is symmetric.

$$\max_x V(x, x^*) = \max_x \frac{\left([(1-\theta_h)n]^{1-\theta_h}[\theta_f n^*]^{\theta_h}\right)^{1-\sigma} - 1}{1-\sigma} - \chi n^\xi$$

Taking the derivative of V with respect to x gives:

$$\frac{\partial V}{\partial x} = C^{-\sigma}\left[(1-\theta_h)^2\left(\frac{C^f}{C^h}\right)^{\theta_h}\frac{\partial n}{\partial x} + \theta_h\theta_f\left(\frac{C^h}{C^f}\right)^{1-\theta_h}\frac{\partial n^*}{\partial x}\right] - \chi\xi n^{\xi-1}\frac{\partial n}{\partial x}$$

where n, n^*, C^h, C^f, and C are given by (55), (56), (49), (50), and (21), respectively. Setting the derivative equal to zero, it can be rewritten:

$$(1-\theta_h)^\xi \left(C^h\right)^{\Delta_h}\left(C^f\right)^{\Sigma_h}\left[(1-\theta_h) + \theta_h\frac{n\frac{\partial n^*}{\partial x}}{n^*\frac{\partial n}{\partial x}}\right] = \chi\xi$$

where $\Delta_h = (1-\theta_h)(1-\sigma) - \xi$ and $\Sigma_h = \theta_h\,(1-\sigma)$. Writing C^h and C^f in terms of n and n^*, the expression can be rewritten in the following way:

$$(n)^{\Delta_h}\,(n^*)^{\Sigma_h}\left[(1-\theta_h) + \theta_h \frac{n\frac{\partial n^*}{\partial x}}{n^*\frac{\partial n}{\partial x}}\right] = \frac{\chi\xi}{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}(\theta_f)^{\theta_h(1-\sigma)}}$$

The following two expressions are important for finding the solution and for understanding why the optimal monetary policy rules are independent of the policy choice of the other country.

$$\frac{n\frac{\partial n^*}{\partial x}}{n^*\frac{\partial n}{\partial x}} = \frac{-\Sigma_f}{\Delta_f} \quad \text{and} \quad (n)^{\Delta_h}\,(n^*)^{\Sigma_h} = x\Omega_h$$

where $\Delta_f = (1-\theta_f)(1-\sigma) - \xi$, $\Sigma_f = \theta_f\,(1-\sigma)$, and:

$$\Omega_h = \left(\frac{\varepsilon}{\varepsilon - 1}\right)\frac{\chi\xi}{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}(\theta_f)^{\theta_h(1-\sigma)}}$$

So now the optimal money growth rate for the Home country can be written in the following way:

$$\begin{aligned}\hat{x} &= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{\Delta_f}{(1-\theta_h)\Delta_f - \theta_h\Sigma_f} \\ &= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{(1-\theta_f)(1-\sigma)-\xi}{(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_h)\xi}\end{aligned}$$

Using the analogous symmetric value function of the Foreign representative agent, one can solve the Foreign monetary authority's maximization problem and get the symmetric result that the optimal rate of Foreign money growth is given by:

$$\begin{aligned}\hat{x}^* &= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{\Delta_h}{(1-\theta_f)\Delta_h - \theta_f\Sigma_h} \\ &= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{(1-\theta_h)(1-\sigma)-\xi}{(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_f)\xi}\end{aligned}$$

The reason why $\hat{x}$ is not a function of x^* and why $\hat{x}^*$ is not a function of x is because the equilibrium derivative $\frac{\partial U(C)}{\partial x}$ divided by the equilibrium derivative $\frac{\partial g(n)}{\partial x}$ is independent of x^*. This reduces down to the two expressions above for $(n)^{\Delta_h}\,(n^*)^{\Sigma_h}$ and $\frac{n}{n^*}\frac{\partial n^*}{\partial x}\left(\frac{\partial n}{\partial x}\right)^{-1}$. Both of them are independent of x^*.

T-2 Derivation of Demand and Price Equations in Two-Country OLG Model with Imperfectly Competitive Labor Supply

In this section, I derive the Home-country individual demand equations for Home consumption c^h_{t+1} and Foreign consumption c^f_{t+1}, as well as a Home-country consumer price level P_{t+1} that incorporates the prices of both Home and Foreign goods as shown in equation (14) and its Foreign country analogue. The derivation for the analogous Foreign-country demand functions and consumer price level are symmetric. I use an individual cost-minimization problem to derive the demand functions—even though the same functions result from utility maximization—because the cost-minimization problem provides an intuitive solution for the consumer price level.

From the lifetime utility function in equation (20), individual agents only care about some index of aggregate consumption. The following is a general specification of a CES consumption aggregator:

$$c_{t+1} \equiv \left[(1-\theta_h)^{\frac{1}{\rho}}\left(c^h_{t+1}\right)^{\frac{\rho-1}{\rho}} + \theta_h^{\frac{1}{\rho}}\left(c^f_{t+1}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \quad \forall t$$
$$\text{where} \quad \theta_h \in \left[0, \frac{1}{2}\right] \quad \text{and} \quad \rho \in [0, \varepsilon) \tag{T.2.1}$$

where ρ is the elasticity of substitution between a unit of Home-produced good and a unit of Foreign-produced good, and θ^h is the home bias parameter that characterizes the degree of openness of the country. The exponent on the home-bias parameter $1/\rho$ follows the Armington aggregator form.

The Home-agent individual demand functions for consumption of Home-produced good c^h_{t+1} and Foreign-produced good c^f_{t+1} can be derived by solving the problem of the individual choosing how much of each type of good to consume, given the prices of each type of good P^h_{t+1} and P^f_{t+1} and given a particular level of aggregate consumption c_{t+1}, in order to minimize total expenditures.[21]

$$\min_{c^h_{t+1}, c^f_{t+1}} \; P^h_{t+1}c^h_{t+1} + e_t P^f_{t+1}c^f_{t+1} \quad \text{s.t.} \quad c_{t+1} \leq \left[(1-\theta)^{\frac{1}{\rho}}\left(c^h_{t+1}\right)^{\frac{\rho-1}{\rho}} + \theta^{\frac{1}{\rho}}\left(c^f_{t+1}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{T.2.2}$$

The Lagrangian for this problem is:

$$\mathcal{L} = P^h_{t+1}c^h_{t+1} + e_t P^f_{t+1}c^f_{t+1} + P_{t+1}\left(c_{t+1} - \left[(1-\theta)^{\frac{1}{\rho}}\left(c^h_{t+1}\right)^{\frac{\rho-1}{\rho}} + \theta^{\frac{1}{\rho}}\left(c^f_{t+1}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}\right) \tag{T.2.3}$$

where P_{t+1} is the multiplier on the constraint and represents the marginal cost of an extra unit of aggregate consumption. So P_{t+1} is interpreted as the price of aggregate

[21] The dual problem of maximizing the level of aggregate consumption subject to a budget constraint of expenditures being less than the currency held at the time of exchange does not yield the same result because the multiplier on the budget constraint does not have the interpretation as the price of an extra unit of aggregate consumption.

consumption. The first order conditions are the following:

$$P^h_{t+1} = P_{t+1}\left[\frac{(1-\theta)c_{t+1}}{c^h_{t+1}}\right]^{\frac{1}{\rho}} \tag{T.2.4}$$

$$e_t P^f_{t+1} = P_{t+1}\left[\frac{\theta c_{t+1}}{c^f_{t+1}}\right]^{\frac{1}{\rho}} \tag{T.2.5}$$

$$c_{t+1} = \left[(1-\theta)^{\frac{1}{\rho}}\left(c^h_{t+1}\right)^{\frac{\rho-1}{\rho}} + \theta^{\frac{1}{\rho}}\left(c^f_{t+1}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{T.2.6}$$

Dividing (T.2.4) by (T.2.5) gives the following relationship:

$$\frac{P^h_{t+1}}{e_t P^f_{t+1}}\left(\frac{c^h_{t+1}}{c^f_{t+1}}\right)^{\frac{1}{\rho}} = \left(\frac{1-\theta}{\theta}\right)^{\frac{1}{\rho}} \tag{T.2.7}$$

Notice that in the Cobb-Douglas or log utility case when $\rho = 1$, the ratio of Home consumption expenditure to Foreign consumption expenditure is a constant.[22] Also, note that solving (T.2.4) and (T.2.5) for c^h_{t+1} and c^f_{t+1}, respectively, gives Home demand equations for aggregate consumption of Home goods and aggregate consumption of Foreign goods.

$$c^h_{t+1} = (1-\theta)\left(\frac{P^h_{t+1}}{P_{t+1}}\right)^{-\rho} c_{t+1} \tag{T.2.8}$$

$$c^f_{t+1} = \theta\left(\frac{e_t P^f_{t+1}}{P_{t+1}}\right)^{-\rho} c_{t+1} \tag{T.2.9}$$

These demand equations are analogous to the differentiated labor input demand equations in (13), except that they include the home-bias parameter.

The expression for the aggregate price index P_{t+1} of the Home consumption over aggregate Home and Foreign consumption is found by rewriting (T.2.6) as:

$$c_{t+1} = \left[\left(\frac{1-\theta}{c^h_{t+1}}\right)^{\frac{1}{\rho}} c^h_{t+1} + \left(\frac{\theta}{c^f_{t+1}}\right)^{\frac{1}{\rho}} c^f_{t+1}\right]^{\frac{\rho}{\rho-1}} \tag{T.2.10}$$

Then, substituting the expressions for $\left([1-\theta]/c^h_{t+1}\right)^{1/\rho}$ and $\left(\theta/c^f_{t+1}\right)^{1/\rho}$ from (T.2.4) and (T.2.5) into (T.2.10) gives the expression for aggregate expenditures which is implied by the cost minimization problem in (T.2.2):

$$P_{t+1}c_{t+1} = P^h_{t+1}c^h_{t+1} + e_t P^f_{t+1}c^f_{t+1} \tag{T.2.11}$$

[22] The outcome when $\rho = 1$ is different from the case when $\rho \in [0, \varepsilon)$ and $\rho \neq 1$ in a critical way. As is shown in Appendix T-4, in the case when $\rho \neq 1$, an international Bertrand duopoly situation develops between monetary authorities, and the world equilibrium is (x, x^*). [*This last sentence may not be correct.*]

Now divide (T.2.11) by aggregate consumption c_{t+1} and plug in the expressions for c^h_{t+1}/c_{t+1} and c^f_{t+1}/c_{t+1} from (T.2.4) and (T.2.5). The resulting expression for aggregate price is:

$$P_{t+1} = \left[(1-\theta)\left(P^h_{t+1}\right)^{1-\rho} + \theta\left(e_t P^f_{t+1}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{T.2.12}$$

Note that this expression is the Home country CPI and is analogous to the within-country price aggregator in equation (14) but with the inclusion of the Home-bias parameter θ.

In the case of Cobb-Douglas aggregation over aggregate Home consumption and aggregate Foreign consumption ($\rho = 1$), the expression for aggregate price is:

$$P_{t+1} = \frac{1}{(1-\theta)^{1-\theta}\theta^\theta}\left(P^h_{t+1}\right)^{1-\theta}\left(e_t P^f_{t+1}\right)^\theta \tag{T.2.13}$$

and total aggregate expenditure is given by:

$$P_{t+1}c_{t+1} = \frac{1}{(1-\theta)^{1-\theta}\theta^\theta}\left(P^h_{t+1}c^h_{t+1}\right)^{1-\theta}\left(e_t P^f_{t+1}c^f_{t+1}\right)^\theta \tag{T.2.14}$$

T-3 Properties of International Model CES aggregator

In this paper, I assume a specific case of the general CES functional form for aggregate consumption of a given Home or Foreign consumer. As defined in Section 2.2, the CES production technology of the representative firm in the Home and Foreign countries are, respectively:

$$y_t \equiv \left(\int_0^1 n_t(z)^{\frac{\varepsilon-1}{\varepsilon}}\, dz\right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall\, t \tag{T.3.1}$$

$$y_t^* \equiv \left(\int_0^1 n_t(z^*)^{\frac{\varepsilon-1}{\varepsilon}}\, dz^*\right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall\, t \tag{T.3.2}$$

where $\varepsilon \geq 1$ is the constant elasticity of substitution among differentiated labor inputs in either the Home or Foreign country. The aggregator over both Home aggregate consumption c^h_{t+1} and aggregate Foreign consumption c^f_{t+1} takes the same general CES form as in (T.3.2).

$$C_{t+1} \equiv \left[(1-\theta)^{\frac{1}{\rho}}\left(C^h_{t+1}\right)^{\frac{\rho-1}{\rho}} + \theta^{\frac{1}{\rho}}\left(C^f_{t+1}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \quad \text{for} \quad \theta \in \left[0, \frac{1}{2}\right] \tag{T.3.3}$$

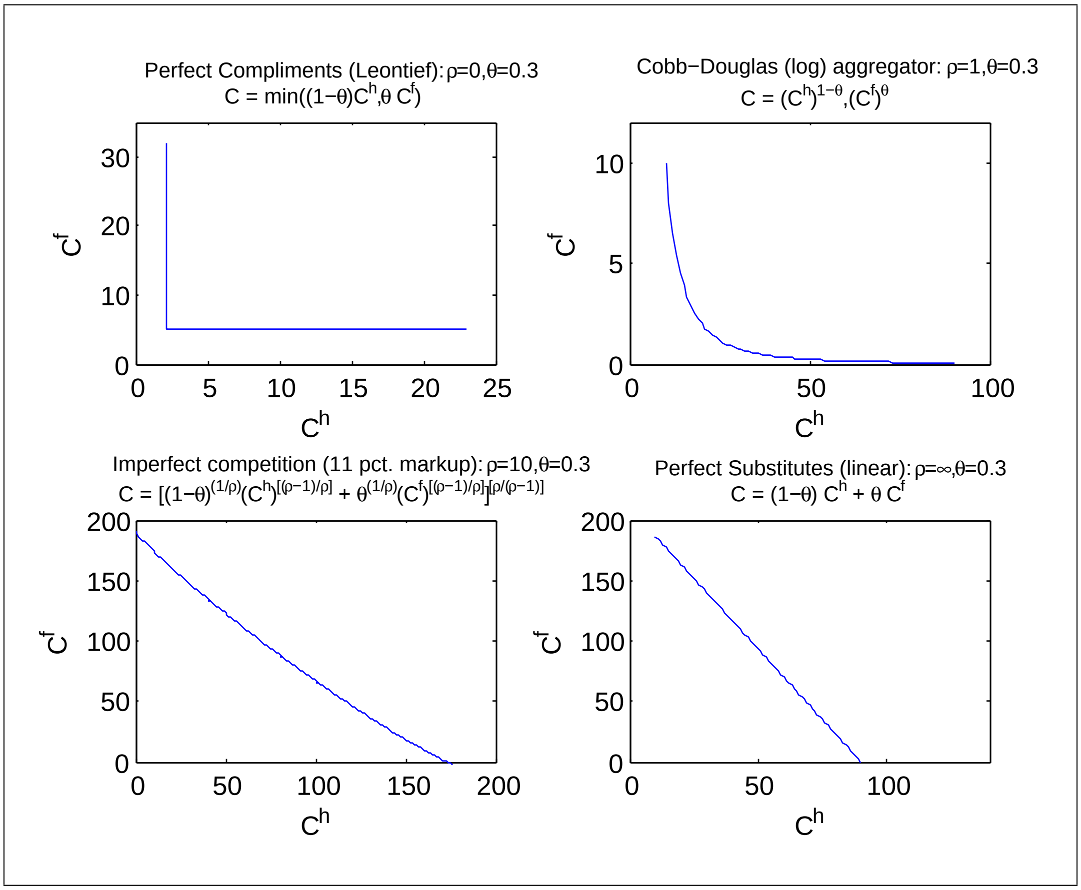
where θ is a Home-bias parameter and $\rho \in [0, \varepsilon)$ is the elasticity of substitution between a unit of Home consumption and a unit of Foreign consumption. The only restriction is that the elasticity of substitution between aggregate Home consumption and aggregate Foreign consumption is assumed to be less than or equal to the elasticity of substitution among the differentiated goods in either country ($\rho \leq \varepsilon$). In the analyses in Section 2, I assume a specific case of (T.3.3) in which the aggregator assumes a Cobb-Douglas form ($\rho = 1$). The general CES aggregator is an attractive form because it nests so many economically relevant cases.

Figure 6 shows various specifications of the general CES aggregator function in (T.3.3). Taking the limit of (T.3.3) as $\rho \rightarrow 0$, a fixed level of aggregate consumption takes the Leontief form of perfect complements as shown in the first panel of Figure 6. Using L'Hospital's rule when taking the limit of (T.3.3) as $\rho \rightarrow 1$, the aggregator function corresponding to unit elasticity ($\rho = 1$) is Cobb-Douglas or log linear utility as shown in the second panel in Figure 6. Lastly, the fourth panel shows that the linear aggregator (perfect substitutes or perfect competition) is the resulting aggregator function as $\rho \rightarrow \infty$. This reflects the case of perfect competition. Included in the third panel of Figure 6 shows the shape of the general CES aggregator function when the elasticity of substitution is at its often calibrated value of 10.

The key result here is that each constant elasticity consumption aggregator or utility curve becomes flatter as the elasticity of substitution increases from the perfectly inelastic case of $\rho = 0$ to the perfectly elastic case of $\rho = \infty$. The exponent of $1/\rho$ on the Home bias terms is merely a convenience to make the resulting constant expenditure ratio a more simple expression.

It is important, however, to recognize that the common assumption of a logarithmic or Cobb-Douglas aggregator is implicitly assuming a unit elasticity of substitution

Figure 6: Various specifications of general CES aggregator function



between Home and Foreign aggregate consumption. As is shown in equation (29) of Section 2.3, the case of $\rho = 1$ implicitly makes the strong assumption that individuals exchange a constant share of their revenues for Foreign currency. Appendix T-4 addresses the solution to the model when the total consumption aggregator takes its general form ($\rho \geq 0$).

T-4 Solutions for General CES Aggregator

The purpose of this appendix is to document some of the solutions to the equilibrium problem when the CES aggregator over Home aggregate consumption is not restricted to the Cobb-Douglas case of unit elasticity of substitution ($\rho \neq 1$).

$$c_{t+1} \equiv \left[(1-\theta_h)^{\frac{1}{\rho}} \left(c_{t+1}^h\right)^{\frac{\rho-1}{\rho}} + \theta_h^{\frac{1}{\rho}} \left(c_{t+1}^f\right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \quad \text{for} \quad \theta_h \in \left(0, \frac{1}{2}\right] \tag{T.4.1}$$

The maximization problem analogous to (28) is the following:

$$\max_{m_t^f, w_t(z)} \frac{\left(\left[(1-\theta_h)^{\frac{1}{\rho}} \left(\frac{w_t(z)^{1-\varepsilon}}{P_{t+1}^h \left(P_t^h\right)^{-\varepsilon}} y_t - \frac{e_t m_t^f - (x-1)xM_{t-1}}{P_{t+1}^h} \right)^{\frac{\rho-1}{\rho}} + \theta_h^{\frac{1}{\rho}} \left(\frac{m_t^f}{P_{t+1}^f} \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \right)^{1-\sigma} - 1}{1-\sigma} \cdots$$
$$-\chi \left[\left(\frac{w_t(z)}{P_t^h} \right)^{-\varepsilon} y_t \right]^{\xi} \tag{T.4.2}$$

where the two first order conditions, analogous to (29) and (30), are:

$$\frac{P_{t+1}^h}{e_t P_{t+1}^f} \left(\frac{c_{t+1}^h}{c_{t+1}^f} \right)^{\frac{1}{\rho}} = \left(\frac{1-\theta_h}{\theta_h} \right)^{\frac{1}{\rho}} \tag{T.4.3}$$

$$(1-\theta_h)^{\frac{1}{\rho}} \left(\frac{\varepsilon - 1}{\varepsilon} \right) \frac{w_t(z)}{P_{t+1}^h} \left(c_{t+1}\right)^{\frac{1}{\rho}-\sigma} \left(c_{t+1}^h\right)^{-\frac{1}{\rho}} = \chi\xi \left(n_t(z)\right)^{\xi-1} \tag{T.4.4}$$

where equation (T.4.3) equates the marginal cost of giving up a Home-currency unit of Home consumption for the marginal benefit of a Home-currency unit of Foreign consumption. Equation (T.4.4) equates the real wage with the marginal rate of substitution between consumption and leisure. The market clearing conditions are the same as in Section 2.4. The equations that characterize an equilibrium in this case, given monetary policy (x, x^*) are shown in Table 6.

The steady state equilibrium inflation rates are again equal to the money growth rates as in (45) and (46). However, because the first order condition for m_t^f in (T.4.3) no longer implies a constant expenditure share on Home and Foreign consumption, in general, individuals can substitute away from Foreign expenditure when the inflation tax of the Foreign country's monetary policy adversely affects them. A key point here is that, when $\rho = 1$ and the aggregator is Cobb-Douglas, agents are bound to hold a specific fraction of their revenues in Foreign currency. Thus, $\rho = 1$ renders the demand for Foreign currency inelastic. When $\rho \neq 1$ the elasticity of demand for Foreign currency becomes elastic.

As in Section 2.5, the steady state equilibrium inflation level is found by substituting the money market clearing conditions (33) and (34) and the currency exchange

Table 6: Equilibrium conditions given x and x^* with general CES aggregator

	Home country	Foreign country
(29′)	$\frac{P_{t+1}^h}{e_t P_{t+1}^f}\left(\frac{c_{t+1}^h}{c_{t+1}^f}\right)^{\frac{1}{\rho}} = \left(\frac{1-\theta_h}{\theta_h}\right)^{\frac{1}{\rho}}$	$\frac{e_t P_{t+1}^f}{P_{t+1}^h}\left(\frac{c_{t+1}^{f*}}{c_{t+1}^{h*}}\right)^{\frac{1}{\rho}} = \left(\frac{1-\theta_f}{\theta_f}\right)^{\frac{1}{\rho}}$
(30′)	$(1-\theta_h)^{\frac{1}{\rho}}\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{w_t(z)}{P_{t+1}^h}\frac{\left(c_{t+1}\right)^{\frac{1}{\rho}-\sigma}}{\left(c_{t+1}^h\right)^{\frac{1}{\rho}}} = \chi\xi\left(n_t(z)\right)^{\xi-1}$	$(1-\theta_f)^{\frac{1}{\rho}}\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{w_t(z^*)}{P_{t+1}^f}\frac{\left(c_{t+1}^*\right)^{\frac{1}{\rho}-\sigma}}{\left(c_{t+1}^{f*}\right)^{\frac{1}{\rho}}} = \chi\xi\left(n_t(z^*)\right)^{\xi-1}$
(13)	$n_t(z) = \left(\frac{w_t(z)}{P_t^h}\right)^{-\varepsilon} y_t$	$n_t(z^*) = \left(\frac{w_t(z^*)}{P_t^f}\right)^{-\varepsilon} y_t^*$
(17)	$w_t(z)n_t(z) = m_t^h + e_t m_t^f$	$w_t(z^*)n_t(z^*) = m_t^{f*} + \frac{m_t^{h*}}{e_t}$
(25)	$c_{t+1}^h = \frac{m_t^h+(x-1)xM_{t-1}}{P_{t+1}^h}$	$c_{t+1}^{f*} = \frac{m_t^{f*}+(x^*-1)x^*M_{t-1}^*}{P_{t+1}^f}$
(26)	$c_{t+1}^f = \frac{m_t^f}{P_{t+1}^f}$	$c_{t+1}^{h*} = \frac{m_t^{h*}}{P_{t+1}^h}$
(21′)	$c_{t+1} = \left[(1-\theta_h)^{\frac{1}{\rho}}\left(c_{t+1}^h\right)^{\frac{\rho-1}{\rho}} + \theta_h^{\frac{1}{\rho}}\left(c_{t+1}^f\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$	$c_{t+1}^* = \left[(1-\theta_f)^{\frac{1}{\rho}}\left(c_{t+1}^{f*}\right)^{\frac{\rho-1}{\rho}} + \theta_f^{\frac{1}{\rho}}\left(c_{t+1}^{h*}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$
	Market clearing conditions	
(31)	$y_t = c_t^h + c_t^{h*}$	
(32)	$y_t^* = c_t^f + c_t^{f*}$	
(33)	$M_t = m_t^h + m_t^{h*}$	
(34)	$M_t^* = m_t^f + m_t^{f*}$	
(35)	$e_t m_t^f = m_t^{h*}$	

market clearing condition (35) into the portfolio constraint (17) and its Foreign analogue, and then iterating the constraint one period forward.

$$\frac{P_{t+1}^h}{P_t^h} = x \tag{T.4.5}$$

$$\frac{P_{t+1}^f}{P_t^f} = x^* \tag{T.4.6}$$

The steady state equilibrium exchange rate is found by plugging the expressions for the currency shares from (37) and (38) into the currency exchange market clearing condition (35).

$$e_t = \frac{(1-\phi)M_t}{(1-\phi^*)M_t^*} \tag{T.4.7}$$

Plugging in the expressions for the currency shares from (37) and (38), the equilibrium inflation rates (T.4.5) and (T.4.6), and using the currency exchange market clearing condition (35), the expressions for steady-state equilibrium aggregate consumption levels given x and x^* are the following:

$$c^h = \frac{(\phi + x - 1)n}{x} \tag{T.4.8}$$

$$c^f = \frac{(1-\phi^*)n^*}{x^*} \tag{T.4.9}$$

$$c^{f*} = \frac{(\phi^* + x^* - 1)n^*}{x^*} \tag{T.4.10}$$

$$c^{h^*} = \frac{(1-\phi)n}{x} \tag{T.4.11}$$

and the steady state equilibrium expressions for Home and Foreign employment are:

$$n = \frac{xM_{t-1}}{w_t(z)} \tag{T.4.12}$$

$$n^* = \frac{x^*M^*_{t-1}}{w_t(z^*)} \tag{T.4.13}$$

Now taking the steady state equilibrium values of (T.4.8) through (T.4.13) as well as the equilibrium characterizations for prices and the exchange rate from (T.4.5), (T.4.6), and (T.4.7), and substituting them into the two first order conditions for the Home country (T.4.3) and (T.4.4) and their two Foreign analogues, the steady state equilibrium is characterized by the following set of four equations in four unknowns $(\phi, w_t(z), \phi^*, w_t(z^*))$:

$$c^h \left(c^f\right)^{\rho-1} \left(c^{h*}\right)^{-\rho} = \frac{1-\theta_h}{\theta_h} \tag{T.4.14}$$

$$c^{f*} \left(c^{h*}\right)^{\rho-1} \left(c^f\right)^{-\rho} = \frac{1-\theta_f}{\theta_f} \tag{T.4.15}$$

$$\frac{(1-\theta_h)^{\frac{1}{\rho}}}{x} \left(\frac{\varepsilon-1}{\varepsilon}\right) \frac{\left(c\right)^{\frac{1}{\rho}-\sigma}}{\left(c^h\right)^{\frac{1}{\rho}}} = \chi\xi \left(\frac{xM_{t-1}}{w_t(z)}\right)^{\xi-1} \tag{T.4.16}$$

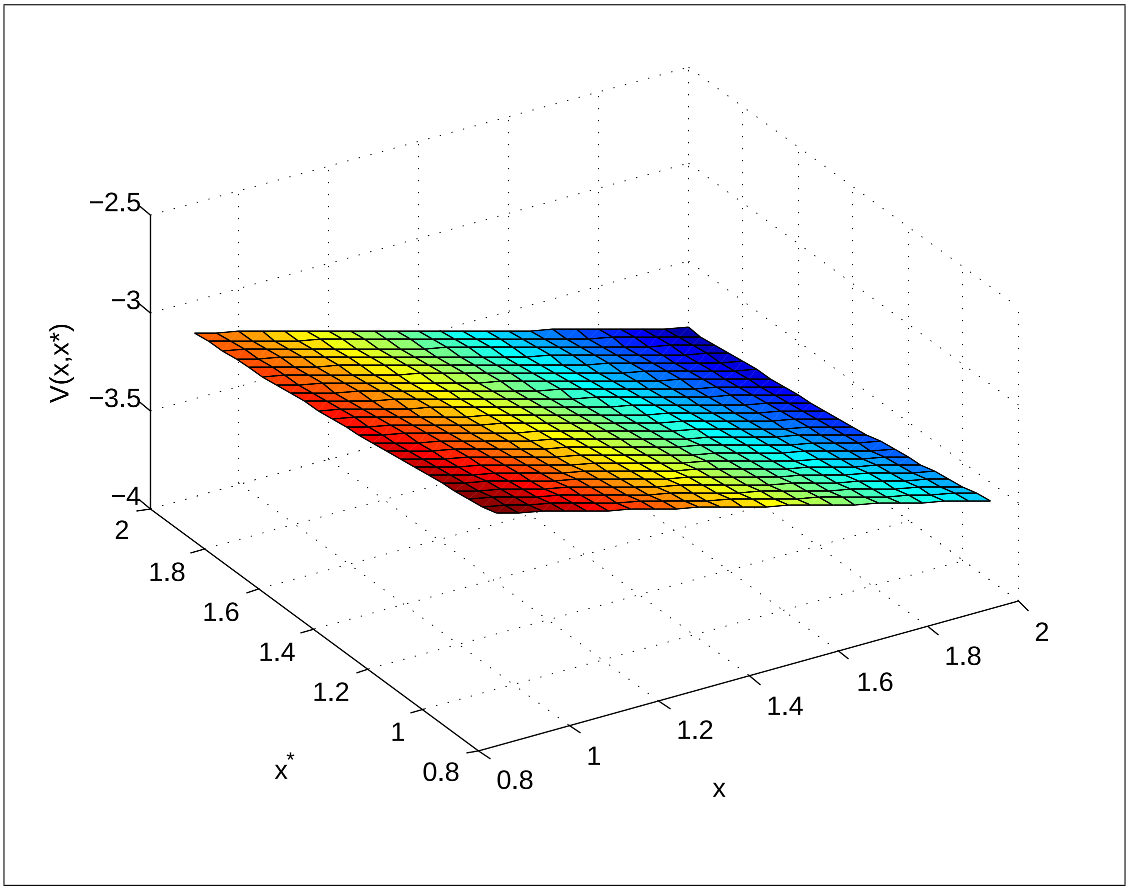
$$\frac{(1-\theta_f)^{\frac{1}{\rho}}}{x^*} \left(\frac{\varepsilon-1}{\varepsilon}\right) \frac{\left(c^*\right)^{\frac{1}{\rho}-\sigma}}{\left(c^{f*}\right)^{\frac{1}{\rho}}} = \chi\xi \left(\frac{x^*M^*_{t-1}}{w_t(z^*)}\right)^{\xi-1} \tag{T.4.17}$$

Thus, the policy functions are functions of the parameters, x and x^*, and state variables M_{t-1} and M^*_{t-1}. The state variables are normalized to 1 in this case without loss of generality.

Because the model with the general CES aggregator ($0 < \rho < \infty$ and $\rho \neq 1$) has no analytical solution, I solve it numerically.[23] Figure 7 shows the value function $V(x, x^*)$ of a representative agent in the Home country. It was calibrated such that $\theta = 0.75$, $\sigma = 3$, $\varepsilon = 10$, $\chi = 2$, $\xi = 2$, $\rho = 2$, and $x, x^* \in (0.9, 2.0)$.

[23] I discretize the (x, x^*) state space and use a Nelder-Mead simplex search method to find the solution at each point. The code for this computation is available upon request.

Figure 7: $V(x, x^*)$ **in general CES case**



Calibrated parameters: $\theta_h = \theta_f = 0.75$, $\sigma = 3$, $\varepsilon = 10$, $\chi = 2$, $\xi = 2$, $\rho = 0.95$.

The main difference here from the Cobb-Douglas aggregator case in the paper in which $\rho = 1$ is that the optimal expenditure share on Home currency ϕ is now a function of both x and x^*. So individuals can substitute away from Foreign currency expenditure if it becomes too expensive in terms of Home consumption. This induces an international Bertrand duopoly situation between the two monetary authorities. That is, the lower money growth rate a monetary authority chooses, the more attractive are the terms of trade for a Foreign country. It becomes a race to the bottom and the world equilibrium monetary policy is $(x = x* = 0)$.

T-5 Some Literature Comments

See Evans (2007) for a good survey of the literature. Also Wynne and Kersting (2007) give a good survey.

T-5.1 Barro and Gordon (1983) and Rogoff (1985)

Comparison between this analysis and Rogoff (1985) and Barro and Gordon (1983) (RBG).

- This paper and RBG both have imperfect competition in labor supply (labor unions) causing a wedge between the socially optimal output level and the labor supplied by the unions.
- No wage contracts
- no time consistent policy

Barro and Gordon (1983)

- natural rate models such as Lucas (1972), Sargent and Wallace (1975), and Barro and Gordon (1983) suggest that the anticipated or systematic parts of monetary policy are neutral.
- the parameter $k < 1$ in Barro and Gordon (1983) that characterizes the degree to which the target level of unemployment (k times the natural rate U^n) could be caused by unemployment compensation, income taxation, or imperfect competition in labor supply (unions).
- inflation being able to cause employment to deviate from natural rate implies price stickiness and assumes that suppliers always meet demand.
- Problems
  - Barro and Gordon (1983, p.592) assume that the Phillips curve relationship "reflects the maximizing behavior of private agents on decentralized markets."
  - Azariadis (1981, p. 947) criticized the natural rate Phillips curve with the following statement. "To convince our impartial observer that [the natural rate Phillips curve] is valid economic theorizing, we shall have to show him that it is derived from first principles.... [The natural rate Phillips curve] is then no more than the first two terms from the Taylor-series expansion of the 'true' aggregate supply schedule around $E\log p$. We have no assurance that this infinite series converges fast enough to justify dropping terms of higher order, or that [the natural rate Phillips curve] retains all the qualitative properties of the true supply schedule. ...First-order series expansions, for instance, do not retain the curvature properties of the approximand."

- Why does can societal welfare be represented by a quadratic loss or cost function.

- Woodford (2003, p. 57) begins a summary of the derivation of the quadratic loss function saying, "a quadratic approximation to expected utility, which suffices (under certain conditions) for the derivation of a linear approximation to an optimal policy rule, can be expressed in terms of the expected value of squared deviations of certain aggregate variables from target values for those variables...."

- partially indexed one period nominal wage contracts negotiated by unions form the friction that allows money to be non-neutral

T-5.2 The Current Account

Because the model has no capital investment or government spending, the current account is simply $CA_t = C_t^{h*} - C_t^f = EX_t - IM_t = \frac{m_{t-1}^{h*}}{P_t^h} + \frac{m_{t-1}^f}{P_t^f}$. So currency is the only form of debt, or claims on future consumption, and the current account represents the net real indebtedness of the country.

T-5.3 Obstfeld and Rogoff (1996)

- Obstfeld and Rogoff (1996, p. 199) Two important relative prices: the *real exchange rate* and the *terms of trade*. "Both of these relative prices play central roles... in a open economy's adjustment to economic shocks."
  - real exchange rate is $\frac{P}{P^*}$, where P is the price index of the Home country and P^* is the price index of the Foreign country.
  - the terms of trade is the relative price of exports in terms of imports $\frac{P^h}{eP^f}$, where P^h is the price index of Home-produced goods, P^f is the price index of Foreign-produced goods, and e is the exchange rate

- absolute purchasing power parity: the *real exchange rate* always tends to 1.

- relative purchasing power parity: changes in national price levels tend to be equal, or changes in the *real exchange rate* tend to be equal.

- Obstfeld and Rogoff (1996, pp. 595-97) have a two-country, two CIA constraint model setup similar to what I have done. But they don't take it past the first order conditions.

- Obstfeld and Rogoff (1996, p.635) note that my type of model actually has price stickiness built in because whoever has the imperfect competition is forced to supply at the demand curve. This is consistent with the one-period wage or price contracts story.

- Obstfeld and Rogoff (1996, pp. 648-57) has a derivation of the simple international version of Barro and Gordon (1983). They mention that this model simply assumes the aggregate supply relationship (Phillips curve) but that its feature of target output deviating from the natural rate could come from income taxes, minimum-wage laws, or imperfect competition on the part of labor suppliers. Pages 653-54 treat the question of openness and inflation, specifically. Obstfeld and Rogoff (1996, pp. 674-75) give a good intuitive justification for using menu costs as the source of sticky prices and why output becomes demand driven in the short run. Obstfeld and Rogoff (1996, p. 676) give a brief summary of the evidence on sticky prices for both wages and producer prices. At the end of this discussion, they state "We do not deny the importance of introducing richer models of price rigidities, and this is one important area where the models of this chapter might be advanced in the future."

T-5.4 Cross country empirical imperfect competition (labor and production) comparisons

Look at Rogoff (2003) for information on imperfect competition levels across countries.

T-5.5 Key motivation: MFD models vs. NOEM

A key motivation of this paper's structural approach to answering the question of how openness affects inflation is that the theoretical models that follow the Mundell-Fleming-Dornbusch type models (hereafter MFD) all predict that a monetary authority setting policy optimally would have an incentive to inflate in a closed economy, but that the inflationary incentive decreases as the country becomes more open to trade.[24]

The MFD models assume a market friction that creates a wedge between the level of employment that a planner would choose $\tilde{n}$ and the level of employment that a wage-setter would choose $\bar{n}$ where $\tilde{n} > \bar{n}$ or rather $\bar{n} = k\tilde{n}$ where $k \in (0, 1)$. The cause of the inefficiency that renders $k < 1$ could be imperfect competition on the part of suppliers of labor or on the part of goods producers, an income tax, unemployment benefits, or a minimum-wage policy.

However, the key characteristic of the MFD models that causes the optimal monetary policy in a closed economy to be an inflationary intervention and is their assumption of a monetary authority's objective function being a quadratic loss function of the following form:

$$L_t = (n_t - \tilde{n}_t)^2 + (\pi_t - \tilde{\pi}_t)^2 \tag{T.5.18}$$

where $\tilde{n} = \frac{\bar{n}_t}{k}$ is the socially optimal employment level and $\tilde{\pi}_t$ is a socially optimal inflation target that incorporates the prices of imports. Also, most MFD and NOEM

[24] A leading example of an MFD model is Rogoff (1985) whose work is an international parallel of the closed economy monetary model of Barro and Gordon (1983). Also, see Obstfeld and Rogoff (1996, ch. 9).

models assume that the prices that are set with market power (either wages or goods prices) are fixed for one period and that suppliers must meet demand. Because the monetary authority's objective is (T.5.18), the monetary authority has an incentive each period to inflate above the level that would induce the owners of labor to supply their optimal level $\bar{n} < \tilde{n}$. Thus, owners of labor adjust their expectations of inflation higher (rational expectations). The result is that equilibrium labor supply remains inefficiently low $\bar{n}$ but equilibrium inflation is high. In an open economy the inflationary incentive is mitigated by the fact that unilateral inflationary policy causes the real interest rate to deteriorate, thereby adding an extra cost to inflation and causing the optimal open-economy level to be lower than its closed economy counterpart.

As mentioned earlier, this result is mostly attributable to the assumed quadratic loss form of the objective function in (T.5.18). Woodford (2003, pp. 383-92) shows that the quadratic loss function in (T.5.18) can be equivalent to a second order Taylor series approximation to expected utility which gives a linear approximation to a policy rule. However, Azariadis (1981) argues that the conditions that make (T.5.18) an appropriate approximation to expected utility are too strong.

Providing support for the skepticism of the quadratic loss approach of the MFD literature is the finding from the NOEM literature that a monetary authority who maximizes the expected utility of a representative consumer in the same type of environment, but without any approximations, faces a deflationary incentive in a closed economy. This is the opposite prediction of the MFD literature. The two types of models share the characteristic that equilibrium employment is below the socially optimal level. The NOEM literature usually motivates this characteristic by using monopolistic competition among goods producers. The incentive of a monetary authority in a closed economy in the NOEM-type environment is to offset the inefficiently high prices resulting from the imperfect competition with a deflationary monetary policy. Deflating actually increases the steady state real wage and causes the equilibrium employment to increase to its socially optimal level.

T-6 Comments

- Replicate Rogoff (1985) so I can explain where he is getting his stuff and what it means.
  - He doesn't derive the Phillips curve relationship for aggregate supply.
  - He doesn't derive the demand function.
  - He doesn't give preferences over home and foreign goods.
- Try to nest Rogoff (1985) in my model (e.g., time consistent monetary policy, Home and Foreign goods perfect substitutes).
  - This entails solving Rogoff (1985).
- See if asymmetric elasticities of substitution $\varepsilon \neq \varepsilon^*$ creates any changes in the results. Because the imperfect competition data reflect differing markups across countries.
  - Hubert Kempf suggested introducing shocks and then test the utility gain from the case of asymmetric variance with symmetric ε, then test the utility gain from the case of symmetric variance with asymmetric ε.
- Show the frictions caused by monetary policy, domestic imperfect competition, and international imperfect substitution like Arseneau (2007) does as driving a wedge between the marginal rate of substitution, the real wage, and the marginal product of labor.
- Must relate this model to the popular framework in Obstfeld and Rogoff (1996, ch. 10).
- Look at results from Goldberg and Tille (2008) that have a three country version of Obstfeld and Rogoff (1995) that looks at the beggar-thy-neighbor effect when other countries (peripheral countries) conduct their transactions in dollars.