Expectations, Open Economies, and Time-consistent Monetary Policy *

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Abstract

This paper asks the question of how domestic monetary authority should optimally respond to foreign monetary policy. I use a dynamic general equilibrium (DGE) model derived from microeconomic foundations in which each country's monetary authority chooses the money growth rate with discretion in order to maximize the welfare of its own citizens. This question has proven difficult for two main reasons. Micro-founded DGE models in which the monetary authority has discretion have been shown to suffer from multiple equilibria. On the other hand, international DGE models with unique equilibria often result in policy rules which are independent of foreign monetary policy. This paper seeks to study optimal monetary policy in an environment in which the monetary authority has discretion and a unique equilibrium exists that is a function of foreign monetary policy. By relaxing the assumption of rational expectations, this two-country model of discretionary optimal monetary policy delivers policy rules that are functions of foreign monetary policy, are not at the upper bound of inflation, and do not rely on trigger strategies.

keywords: Optimal Monetary Policy; International Monetary Policy; Expectations

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1 Introduction

This paper asks the question of how domestic monetary authority should optimally respond to foreign monetary policy. I approach this task using a dynamic general equilibrium (DGE) model derived from microeconomic foundations in which each country's monetary authority chooses the money growth rate with discretion in order to maximize the welfare of its own citizens. This question has proven difficult for two main reasons. Micro-founded DGE models in which the monetary authority has discretion have been shown to suffer from multiple equilibria. On the other hand, international DGE models in which the monetary authority can commit to a policy generate a unique equilibrium. However, these commitment equilibria are dominant strategy equilibria and are independent of foreign monetary policy. This paper seeks to study optimal monetary policy in an environment in which the monetary authority has discretion a unique equilibrium exists that is a function of foreign monetary policy.

Ireland (1997) defines the entire set of time-consistent or "sustainable" equilibria in a closed economy setting. His paper is a more fully specified version of Barro and Gordon (1983) in terms of individual preferences, production technologies, imperfect competition, and monetary objective . In particular, Ireland shows that a continuum of trigger-strategy reputational equilibria exist in this setting. But, similar to Barro and Gordon (1983), Ireland finds a unique non-reputational time-consistent equilibrium in which the monetary authority pushes inflation to its upper bound. This multiplicity of equilibria can be a problem because it implies either that all observed monetary policy is supported some underlying societal strategic threats or that the optimal solution of any monetary authority is to inflate to the maximum.

Ireland (2000) shows that the multiplicity of reputational equilibria, as well as the autarkic non-reputational equilibrium, is a result of the strong assumption of rational expectations. Ireland (2000) shows that relaxing the rational expectations assumption results in a unique non-autarkic steady-state equilibrium that corresponds to the Friedman Rule. The goal of this paper could be summarized as extending Ireland (2000) to a two-country environment.

A number of authors have studied the best response functions of monetary authorities in fully specified two-country DGE models in which monetary authorities can commit to their policy. Cooper and Kempf (2003), Arseneau (2007), and Evans (2007) all study two-country micro-founded DGE models of optimal monetary policy with commitment, and all find unique dominant strategy equilibria. The problem with this result is that anecdotal evidence suggests that monetary authorities are influenced by the actions of foreign monetary authorities and that strategic actions occur among them.

In order to answer the question of how a monetary authority should optimally respond to foreign monetary policy, I will use a model that is most similar to Arseneau (2007). But I will follow Ireland (2000) in that I will not allow each monetary authority to commit to a policy, and I will relax the assumption of rational expectations.

The results of this two-country model of discretionary optimal monetary policy are policy rules that are a function of foreign monetary policy, are not the upper bound of inflation, and do not rely on trigger strategies. The paper is organized as follows. Section 2 presents the model, expectations, and equilibrium definitions. Section 3 presents some numerical simulations of the time path of monetary policy under various specifications of the model. And Section 4 concludes.

2 Model

The model in this paper is a time-consistent version of Arseneau's (2007) two-country model of monetary policy under commitment. I will follow Ireland (2000) and relax the rational expectations assumption in order to obtain a unique nonautarkic steady-state monetary policy that is time-consistent. The two countries are Home and Foreign, which are names rather than relative terms.

Each country is populated by a unit measure of identical infinitely lived households and a unit measure of differentiated goods firms. Each country also has a benevolent monetary authority that chooses the growth rate of its country's money supply in order to maximize the welfare of its own citizens.

2.1 Money

The monetary authority in each country choose the money growth rate in order to maximize the welfare of the representative household in its respective country. The law of motion for money supply in each country can be represented by the following relationship:

$$M_{t+1}^S = x_t M_t^S \qquad \text{and} \qquad M_{t+1}^{*S} = x_t^* M_t^{*S} \tag{1}$$

where M_{t+1}^S is the money supply at the end of period t, $x_t > 0$ is the gross money growth rate chosen by the monetary authority during period t, and M_t^S is the money supply at the beginning of period t. Variables with stars denote Foreign country variables.

Each monetary authority distributes the change in money supply in each period t through a lump-sum transfer to the representative household in its own country.

$$(x_t - 1)M_t^S \qquad \text{and} \qquad (x_t^* - 1)M_t^{*S} \tag{2}$$

A useful normalization of the initial money stock in each country is $M_0^S = M_0^{*S} = 1$. I assume that Home country households only hold Home currency and Foreign country households only hold Foreign currency.[1]

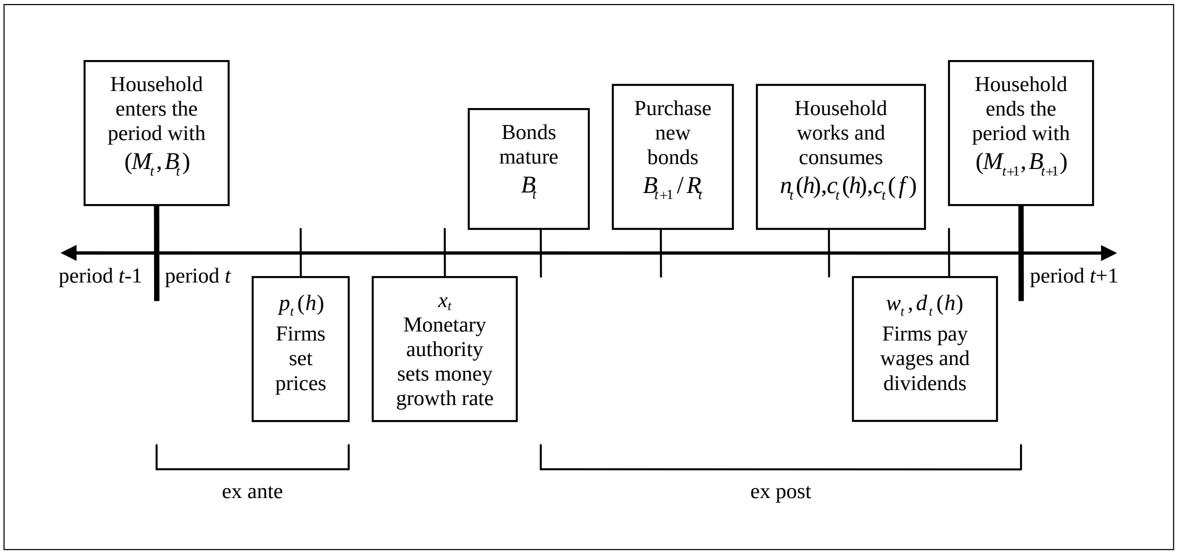
In addition to currency, households in each country can purchase country-specific bonds for B_{t+1}/R_t in period t that return B_{t+1} in period $t+1$. The gross interest rate on bonds purchased in period t is then R_t. This model rules out international trade in bonds, but Arseneau (2007) shows that this constraint does not bind due to the functional form of the model. Other than the bonds and labor, this model assumes no barriers to trade. So the law of one price holds.

[1]This is not a strong assumption, given that currency can be freely exchanged in the currency market. An alternative specification is to allow households to hold both currencies and let the exchange rate be determined by currency exchange market clearing as in Evans (2007).

2.2 Timing

Figure 1 illustrates the timing of the model from the standpoint of the representative household in the Home country. The timing for the Foreign household is symmetric. The key characteristic of the model is that the events in each period can be grouped into pre-monetary decision and post-monetary decision. The firm pricing decision is ex ante, and the rest of the decisions are ex post.

Figure 1: Timing of the model for Home representative agent



The representative household in the Home country enters period t with Home currency balances M_t and unmatured bond holdings B_t from the previous period. The Home household enters the period knowing the prior N periods of money growth rates in both the Home and Foreign countries. So the past data in each household's information set is $\{x_{t-1}, x_{t-2}, ...x_{t-N}\}$ and $\{x^*_{t-1}, x^*_{t-2}, ...x^*_{t-N}\}$.[2]

At the beginning of each period, before the monetary authority chooses a money growth rate, each differentiated goods firm h sets the selling price of its good $p_t(h)$. As will be explained more fully in Section 2.4, a pricing friction is built into the model following Barro and Gordon (1983) in that the ex ante firm price $p_t(h)$ cannot be changed during period t. Once the firm has set its price, the monetary authority sets

[2]I could just as well assume that agents know the entire history of money growth rates, but their expectations are formed using only the prior N periods of data.

the money growth rate for the period x_t and makes a lump-sum transfer $(x_t - 1)M_t^S$ to the representative household of its country.

Once the monetary authority has set the money growth rate x_t, the household's bonds from the previous period mature and the household receives B_t in Home currency. At this point, the household splits into a worker and a shopper. The worker supplies labor to the various firms $n_t(h)$, and the firms produce goods with a linear technology using that labor $y_t(h) = n_t(h)$. The shopper then enters the marketplace and can either buy new bonds B_{t+1} with gross interest rate R_t costing B_{t+1}/R_t or it can purchase consumption of home and foreign differentiated goods, $c_t(h)$ and $c_t(f)$, respectively.

In this model, I assume a cash-in-advance constraint in that all purchases must be paid for in currency. So the cash-in-advance constraint on consumption is the following:

$$M_t + (x_t - 1)M_t^S + B_t - \frac{B_{t+1}}{R_t} \geq P_t c_t \tag{3}$$

where P_t and c_t are aggregations of price and consumption over Home and Foreign prices $[p_t(h), p_t(f)]$ and Home and Foreign consumption $[c_t(h), c_t(f)]$, respectively.

Lastly, after the consumption decisions are made, the firm pays each worker a nominal market wage W_t and pays the representative household a dividend $D_t(h)$. The household then leaves period t with Home currency balances M_{t+1} and bonds to mature next period B_{t+1}. The budget constraint for the representative household is the following:

$$M_t + (x_t - 1)M_t^S + B_t + W_t n_t + \int_0^1 D_t(h)\, dh \geq P_t c_t + \frac{B_{t+1}}{R_t} + M_{t+1} \tag{4}$$

2.3 Households

Each country is populated by a unit measure of identical infinitely lived households. These households maximize lifetime utility by choosing in each period t how much of each differentiated good from each country to consume $c_t(h)$ and $c_t(f)$, how much to work $n_t(h)$, how much wealth to hold in bonds B_{t+1}, and how much wealth to hold in currency M_{t+1}. Household lifetime utility from period t on is given by:

$$U_t = \sum_{j=0}^{\infty} \beta^j \left[ln(c_{t+j}) - \chi n_{t+j} \right] \quad \forall\, t \tag{5}$$

where $\beta \in (0, 1)$ is the discount factor, $\chi > 0$ is a scale parameter in the disutility of labor function, c_t is aggregated consumption over consumption of both Home and Foreign differentiated goods, and n_t is aggregated labor over labor at Home differentiated goods firms. Using the notation of Arseneau (2007) and following the convention of Dixit and Stiglitz (1977), let $c_{H,t}$ be a CES aggregator of Home consumption by Home households, and let $c_{F,t}$ be a CES aggregator of Foreign consumption by Home households given by:

$$c_{H,t} \equiv \left(\int_0^1 c_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{and} \quad c_{F,t} \equiv \left(\int_0^1 c_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall\, t \tag{6}$$

where $\varepsilon > 1$ represents the constant elasticity of substitution between any pair of goods within a country.[3] Then let c_t be defined as the following Cobb-Douglas aggregator over aggregate Home consumption and aggregate Foreign consumption by the Home household:

$$c_t \equiv \left(c_{H,t}\right)^{1-\theta} \left(c_{F,t}\right)^{\theta} \quad \forall\, t \tag{7}$$

where the constant elasticity of substitution between Home and Foreign aggregate consumption is 1, and $\theta \in [0.5, 1]$ is a preference parameter that can be interpreted as the degree of openness of the Home country to Foreign goods.[4] The labor aggregator is defined as the following:

$$n_t \equiv \int_0^1 n_t(h)\, dh \quad \forall\, t \tag{8}$$

The following expressions for household demand for Home and Foreign differenti-

[3] I impose symmetry in the elasticity of substitution ε both within and across countries. It would be an interesting exercise to relax this symmetry.

[4] Evans (2007) shows that θ represents the share of national income spent on imports (import share), which has often been used as a proxy for openness. See Romer (1993) and Wynne and Kersting (2007). Here also I impose symmetry across countries on the degree of openness, so $\theta_h = \theta_f$ and $c_t^* = \left(c_{F,t}^*\right)^{1-\theta} \left(c_{H,t}^*\right)^{\theta}$.

ated goods and within-country aggregated goods can be derived from the definition of the consumption aggregators from (6) and (7) and by solving the household's expenditure minimization problem. The following expressions for aggregate prices also result from this expenditure minimization problem.[5]

$$c_t(h) = \left(\frac{p_t(h)}{p_{H,t}}\right)^{-\varepsilon} c_{H,t} \quad \text{and} \quad c_t(f) = \left(\frac{p_t(f)}{p_{F,t}}\right)^{-\varepsilon} c_{F,t} \tag{9}$$

$$c_{H,t} = (1-\theta)\left(\frac{p_{H,t}}{p_t}\right)^{-1} c_t \quad \text{and} \quad c_{F,t} = \theta\left(\frac{p_{F,t}}{p_t}\right)^{-1} c_t \tag{10}$$

$$p_{H,t} = \left(\int_0^1 p_t(h)^{1-\varepsilon} dh\right)^{\frac{1}{1-\varepsilon}} \quad \text{and} \quad p_{F,t} = \left(\int_0^1 p_t(f)^{1-\varepsilon} df\right)^{\frac{1}{1-\varepsilon}} \tag{11}$$

$$p_t = \frac{1}{\gamma}\left(p_{H,t}\right)^{1-\theta}\left(p_{F,t}\right)^{\theta} \quad \text{where} \quad \gamma = (1-\theta)^{1-\theta}\theta^{\theta} \tag{12}$$

where the lower case letters for the prices above are a normalization by the money supply in period t that will be made clear in the household's utility maximization problem.

Let z_{t+j}^{t} and $z_{t+j}^{*,t}$ represent the representative Home household's expectation with probability 1 of the Home money growth rate x_{t+j} and the Foreign money growth rate x_{t+j}^{*} in period $t+j$ given period t information. Thus, $z_t^t = x_t$ and $z_t^{*,t} = x_t^*$ because the consumer's decision in period t comes after the money growth rates in each country have been established. The exact way in which expectations are formed given past information will be detailed in Section 2.5.

Given the demand functions above, the problem of the representative household is to maximize lifetime utility by choosing aggregate consumption c_t, aggregate labor supply n_t, normalized wealth to hold in bonds b_{t+1}, and normalized wealth to hold in currency m_{t+1}. This problem is given by dividing the cash-in-advance constraint (3) and budget constraint (4) by the Home money supply M_t^S and let lower-case variables represent either real or normalized variables. Also, from this point on, I rewrite the

[5]See Technical Appendix T-1 (available upon request) for the derivation. Note that the demand functions can also be derived from the utility maximization problem, but the derivation of the prices is particularly intuitive in the expenditure minimization problem as they are simply the multipliers on the consumption level constraints.

variables with a superscript that represents the information held by the household.

$$
\begin{aligned}
\max \quad & \sum_{j=0}^{\infty} \beta^j \left[ln(c^t_{t+j}) - \chi n^t_{t+j} \right] \\
& \text{s.t} \quad m^t_{t+j} + z^t_{t+j} - 1 + b^t_{t+j} - \frac{b^t_{t+j+1} z^t_{t+j}}{R^t_{t+j}} \geq p^t_{t+j} c^t_{t+j} \\
& \text{and} \quad m^t_{t+j} + z^t_{t+j} - 1 + b^t_{t+j} + w^t_{t+j} n^t_{t+j} + \int_0^1 d^t_{t+j}(h) dh \geq ... \\
& \qquad p^t_{t+j} c^t_{t+j} + \frac{b^t_{t+j+1} z^t_{t+j}}{R^t_{t+j}} + m^t_{t+j+1} z^t_{t+j} \quad \forall t
\end{aligned} \tag{13}
$$

The solution to this problem, as derived in Appendix A-1, results in the following expressions. The analogous expressions for the Foreign representative household are symmetric.

$$
c^t_t = \frac{x_t}{p^{t-1}_t} \tag{14}
$$

$$
w^t_t = \left(\frac{\chi}{\beta} \right) z^t_t z^t_{t+1} \tag{15}
$$

$$
R^t_t = \frac{z^t_{t+1}}{\beta} \tag{16}
$$

These three equations fully describe household behavior and come out of the household maximization problem.

2.4 Firms

Each country is populated by a unit measure of firms, indexed by $h \in [0, 1]$ in the Home country and $f \in [0, 1]$ in the Foreign country. They each produce a differentiated good, the substitutability of which relative to the other differentiated goods in that country is represented by the households' preference parameter ε. This is the Dixit and Stiglitz (1977) mechanism in which the love of variety on the part of consumers coupled with imperfect substitutability generates an imperfectly competitive market in which firms are able to charge a markup over marginal cost.

Each firm chooses the price of its good $p_t(h)$ at the beginning of the period given its period $t-1$ information, its expectations about period-t what the monetary policy, and its effects on household demand. However, following the New Keynesian price friction laid out by Barro and Gordon (1983), firms cannot change their price after the period-t money growth rate is chosen. Firms must satisfy demand at the given prices, regardless of whether the money growth rate was close to the firms' expectation.

The output of each firm is governed by a linear production technology in labor $y_t(h) = n_t(h)$. Each firm pays the same market wage w_t to its domestic workforce. Again, labor is assumed to not be mobile. Because the firm meets demand in each period given its prices, this implies $y_t(h) = n_t(h) = c_t(h) + c_t^*(h)$. However, the firm must set its ex ante price with period $t-1$ information, before the monetary authorities in each country set their respective money growth rates. So the problem of the domestic firm can be written as:

$$\max_{p_{t+j}(h)} d_{t+j}^{t-1}(h) = \left[p_{t+j}(h) - w_{t+j}^{t-1}\right]\left[c_{t+j}^{t-1}(h) + c_{t+j}^{*,t-1}(h)\right] \tag{17}$$

where $d_{t+j}^{t-1}(h)$ represent the profits of firm h which are paid out as dividends to the representative household in the Home country.[6] Substituting in the expressions for w_{t+j}^{t-1}, $c_{t+j}^{t-1}(h)$, and $c_{t+j}^{*,t-1}(h)$ from Section 2.3 with the information iterated backward one period, the expression for the optimal price level for firm h in period t given period $t-1$ information is the following:

$$p_t^{t-1}(h) = \left(\frac{\varepsilon}{\varepsilon - 1}\right)\left(\frac{\chi}{\beta}\right) z_t^{t-1} z_{t+1}^{t-1} \tag{18}$$

Notice that this means firm price is a markup over the expected wage $p_t^{t-1}(h) = \left(\frac{\varepsilon}{\varepsilon-1}\right) w_t^{t-1}$. Because the right-hand-side of (18) does not depend on h, $p_{H,t}^{t-1} = p_t^{t-1}(h)$.

$$p_{H,t}^{t-1} = p_t^{t-1}(h) = \left(\frac{\varepsilon}{\varepsilon - 1}\right)\left(\frac{\chi}{\beta}\right) z_t^{t-1} z_{t+1}^{t-1} \tag{19}$$

[6] Note that I am assuming that capital markets are closed. Foreign households cannot own stock in Home firms and vice versa.

The symmetric Foreign firm problem produces the following result:

$$p_{F,t}^{*,t-1} = p_t^{*,t-1}(f) = \left(\frac{\varepsilon}{\varepsilon - 1}\right)\left(\frac{\chi}{\beta}\right) z_t^{*,t-1} z_{t+1}^{*,t-1} \tag{20}$$

The remaining prices $p_{F,t}^{t-1}$, $p_{H,t}^{*,t-1}$, p_t^{t-1}, $p_t^{*,t-1}$, are determined by the firm's expectation of the period t exchange rate e_t^{t-1} through the law of one price. The exchange rate is solved for by substituting all four market clearing conditions (30)-(33) described in Section 2.6 into the budget constraint (4).[7]

$$e_{t+j}^{t} = \frac{z_{t+j}^{t}}{z_{t+j}^{*,t}} \tag{21}$$

I follow Arseneau (2007) in assuming that there are no barriers to exchange in currency or goods, so the price of good h in the Home country must equal the price of good h in the Foreign country converted in to Home country units and likewise for the price of the Foreign produced goods.

$$p_t(h) = e_t p_t^*(h) \qquad \text{and} \qquad p_t(f) = e_t p_t^*(f) \tag{22}$$

Given the definitions of the aggregate price indices in (11) and (12), the law of one price at the individual price level (22) implies that the law of one price also holds for the aggregate price indices $p_{H,t} = e_t p_{H,t}^*$, $p_{F,t} = e_t p_{F,t}^*$, and $p_t = e_t p_t^*$. This is a no-arbitrage condition which insures that Home and Foreign households are indifferent between holding either country's currency. It is therefore without loss of generality to make the simplifying assumption that Home households only hold home currency and Foreign households only hold Foreign currency.

Now, using the law of one price (22) and the expression for the expected exchange rate that comes from iterating the information in (21) back one period e_t^{t-1}, the

[7]Appendix A-1 shows that the budget constraint holds with equality.

following expressions result for the remaining price indices:

$$p_{F,t} = \left(\frac{\varepsilon}{\varepsilon - 1}\right)\left(\frac{\chi}{\beta}\right) z_t^{t-1} z_{t+1}^{*,t-1} \tag{23}$$

$$p^*_{H,t} = \left(\frac{\varepsilon}{\varepsilon - 1}\right)\left(\frac{\chi}{\beta}\right) z_t^{*,t-1} z_{t+1}^{*,t-1} \tag{24}$$

$$p_t = \frac{1}{\gamma}\left(\frac{\varepsilon}{\varepsilon - 1}\right)\left(\frac{\chi}{\beta}\right) z_t^{t-1} \left(z_{t+1}^{t-1}\right)^{1-\theta} \left(z_{t+1}^{*,t-1}\right)^{\theta} \tag{25}$$

$$p^*_t = \frac{1}{\gamma}\left(\frac{\varepsilon}{\varepsilon - 1}\right)\left(\frac{\chi}{\beta}\right) z_t^{*,t-1} \left(z_{t+1}^{*,t-1}\right)^{1-\theta} \left(z_{t+1}^{t-1}\right)^{\theta} \tag{26}$$

2.5 Expectations

A large literature has show that the method by which households and firms form their expectations of a policymaker's actions has a profound influence on the policy outcomes. Lucas (1972) showed that allowing agents to consider the policy maker's incentives in a forward-looking manner is a desirable mechanism in economic models. However, Barro and Gordon (1983) and, more recently, Ireland (1997) show that the strong assumption of rational expectations in models of monetary policy leads to a multiplicity of reputational trigger-strategy equilibria as well as a unique highly inflationary steady-state equilibrium.

One problem with the multiple equilibria generated under models of optimal monetary policy in which agents have rational expectations is that even a surprise decrease in the money growth rate will generate a large jump in expected inflation. Ireland (2000) shows in a closed economy model that relaxing the assumption of rational expectations in an intuitive way generates a unique stationary equilibrium that is less inflationary than the rational expectations equilibrium and sometimes equals the deflationary Friedman Rule.

I will impose adaptive expectations functions on this model that follow the same form as in Ireland (2000). Let $z_{t+j}^{t} = \psi_{t+j}^{t}$ and $z_{t+j}^{*,t} = \psi_{t+j}^{*,t}$, where ψ_{t+j}^{t} and $\psi_{t+j}^{*,t}$ are the expectations functions for the money growth rates in the Home country and the Foreign country, respectively, in period $t + j$ given period t information.[8] When

[8] I am assuming here that Home and Foreign households and firms have the same information. That is, the Home households' information about past Foreign money growth rates is the same as

$j = 0$, the expectation function is trivial because the information is known ($\psi_t^t = x_t$ and $\psi_{t+j}^{*,t} = x_t^*$). However, for $j > 0$, the expectations will be functions of $N < \infty$ periods of past money growth rates.

$$z_{t+j}^{t-1} = \psi_{t+j}^{t-1}\left(x_{t-1}, x_{t-2}, ...x_{t-N}\right) \quad \forall t \quad \text{and} \quad j = 0, 1, 2, ... \tag{27}$$

$$z_{t+j}^{*,t-1} = \psi_{t+j}^{*,t-1}\left(x_{t-1}^*, x_{t-2}^*, ...x_{t-N}^*\right) \quad \forall t \quad \text{and} \quad j = 0, 1, 2, ... \tag{28}$$

where the expectations functions are defined on $\psi_{t+j}^{t-1} : R_{++}^N \rightarrow R_{++}$ and $\psi_{t+j}^{*,t-1} : R_{++}^N \rightarrow R_{++}$.

Let ψ represent both ψ_{t+j}^{t-1} and $\psi_{t+j}^{*,t-1}$ for any t or any j. Ireland (2000) proposes the following four intuitive restrictions on the expectations functions ψ.

(R1) ψ is nondecreasing in each of its arguments.
(R2) $\psi\left(x, x, ...x\right) = x$ for all $x \in R_{++}$.
(R3) ψ is continuously differentiable on R_{++}^N.
(R4) $\psi\left(x_1, x_2, ...x_N\right) \geq \beta$ for all $\left(x_1, x_2, ...x_N\right) \in R_{++}^N$ and $x_i \geq \beta$.

Restriction (R1) simply insures that the expected money growth rate moves in the same direction as the actual money growth rate. Restriction (R2) implies that money growth expectations will converge to the actual money growth rate if the rate is held constant long enough. Ireland (2000) notes that (R1) and (R2) allow a monetary authority to build credibility over time. Restriction (R3) limits the extent to which expectations can jump, given any size policy surprise. Lastly, restriction (R4) ensures that a no-arbitrage condition that the interest rate be positive $R_t = z_{t+1}^t/\beta \geq 1$.

Taken together, restriction (R1)-(R4) impose properties on the backward looking expectations ψ that look like some behavior we see in practice. The exact specification of the expectations function used by Ireland (2000) is the following:

$$\psi_{t+j}^{t-1} = \prod_{k=1}^{N} x_{t-k}^{\alpha_k} \quad \forall t \quad \text{and} \quad j = 0, 1, 2, ... \tag{29}$$

where the α_k represent the elasticity of the $t-k$th money growth rate on the expected money growth rate ψ_{t+j}^{t-1} in period $t+j$ given period $t-1$ information. Restriction

Foreign households' information about past Foreign money growth rates. However, this might be an interesting assumption to relax.

(R1) requires that $\alpha_k \geq 0$, and restriction (R2) implies that $\sum_{k=1}^{N} \alpha_k = 1$.

2.6 Equilibrium

This section will define both a private equilibrium in this two-country environment, given expectations and the monetary policy in both countries. I will then define the best response function of the Home monetary authority given the policy of the Foreign monetary authority. The focus on the Home monetary best response function is without loss of generality because the two countries are symmetric.

Four markets must clear in each country in equilibrium. First, the money market must clear in each country, which means that $M_t = M_t^S$. The bond market must clear in both countries, which means a zero net supply. Each goods market must clear, which was already assumed in the firm problem. And lastly, the labor market must clear in both the Home country and the Foreign country.

$$m_t = 1 \quad \text{and} \quad m_t^* = 1 \quad \forall t \tag{30}$$

$$b_t = 0 \quad \text{and} \quad b_t^* = 0 \quad \forall t \tag{31}$$

$$y_t(h) = c_t(h) + c_t^*(h) \quad \text{and} \quad y_t(f) = c_t(f) + c_t^*(f) \quad \forall t, h, f \tag{32}$$

$$n_t(h) = y_t(h) \quad \text{and} \quad n_t(f) = y_t(f) \quad \forall t, h, f \tag{33}$$

I now define a private equilibrium in this two-country economy, given the money growth rates in both the Home and Foreign countries.

Definition 1 (**Private Equilibrium given Home and Foreign Monetary Policy**)**.** A private equilibrium given the money growth rates in the Home and Foreign countries consists of the following:

- expectations $z_{t+j}^{t-1} = \psi_{t+j}^{t-1}$ and $z_{t+j}^{*,t-1} = \psi_{t+j}^{*,t-1}$ for all t and $j = 0, 1, 2, ...$ as shown in (27) and (28) that satisfy restriction (R1)-(R4)
- a sequence of Home money growth rates $\{x_t\}_{t=0}^{\infty}$ and Foreign money growth rates $\{x_t^*\}_{t=0}^{\infty}$ such that $x_i \geq \beta$ and $x_i^* \geq \beta$
- The Home aggregate price index p_t^{t-1} and the Foreign aggregate price index $p_t^{*,t-1}$ are given by (25) and (26), respectively.

- Aggregate consumption by Home households c_t^t and aggregate consumption by Foreign households $c_t^{*,t}$ in period t are given by (14)
- Labor supply in the Home country n_t^t and labor supply in the Foreign country n_t^* are given by (32) and (33) and the expression for consumption demand and their Foreign analogues in (10) and (14)

Before showing the time-consistent problem of the Home monetary authority and its corresponding equilibrium, I will present the commitment equilibrium, which corresponds to the methods and findings of Arseneau (2007). If the Home and Foreign monetary authorities can commit to a money growth rate at the beginning of time, then $x_t = x$ and $x_t^* = x^*$ for all t. The problem of the Home monetary authority is then to choose a constant money growth rate x at $t = 0$ given x^* in order to maximize the welfare of the Home representative household.

$$\begin{aligned} \max_{x} \quad & \sum_{j=t}^{\infty} \beta^t \left[ln(c_t) - \chi n_t \right] \\ \text{s.t} \quad & c_t = \gamma \left(\frac{\varepsilon - 1}{\varepsilon} \right) \left(\frac{\beta}{\chi} \right) (x)^{\theta - 1} (x^*)^{-\theta} \\ \text{and} \quad & n_t = \left(\frac{\varepsilon - 1}{\varepsilon} \right) \left(\frac{\beta}{\chi} \right) \frac{1}{x} \\ \text{and} \quad & R_t \geq 1 \quad \forall t \end{aligned} \tag{34}$$

where the expressions for c_t and n_t in (34) come from their specifications in Definition 1 of the private equilibrium, but with x_t and z_{t+j}^t replaced by x and with x_t^* and $z_{t+j}^{*,t}$ replaced by x^*. A monetary equilibrium with commitment can then be defined as follows.

Definition 2 (**Monetary Nash Equilibrium with Commitment**). A monetary Nash equilibrium with commitment consists of the following:

- All characteristics of Definition 1 of a private equilibrium hold.
- The sequence of Home money growth rates $\{x_t = x\}_{t=0}^{\infty}$ satisfy the maximization problem in (34).
- The sequence of Foreign money growth rates $\{x_t^* = x^*\}_{t=0}^{\infty}$ satisfy the Foreign maximization problem analogous to (34).

- In each period t, the Home money growth rate x is a best response the the Foreign money growth rate, and the Foreign money growth rate x^* is a best response to the Home money growth rate x.

The first step in finding the monetary Nash equilibrium is solving for the best response function that comes out of the maximization problem in (34). The best response function of both the Home and Foreign monetary authorities ends up being a dominant strategy equilibrium. So the best response function also represents the Nash equilibrium. The monetary Nash equilibrium with commitment, shown below, is identical for the Home and Foreign countries because they are symmetric.[9]

$$\hat{x}^c = \hat{x}^{*,c} = \begin{cases} \frac{\beta}{1-\theta}\left(\frac{\varepsilon-1}{\varepsilon}\right) & \text{if} \quad \frac{1}{1-\theta}\left(\frac{\varepsilon-1}{\varepsilon}\right) > 1 \\ \beta & \text{if} \quad \frac{1}{1-\theta}\left(\frac{\varepsilon-1}{\varepsilon}\right) \leq 1 \end{cases} \tag{35}$$

Lastly I define a time-consistent Home Monetary equilibrium given Foreign money growth as a best response function to x^*. The Home monetary authority's maximization problem in the time consistent case is to choose x_t in every period to maximize the lifetime utility of the Home representative household.

$$\begin{aligned} \max_{x_t} \quad & \sum_{j=0}^{\infty} \beta^{t+j}\left[ln(c^t_{t+j}) - \chi n^t_{t+j}\right] \\ \text{s.t} \quad & c^t_t = x_t \left(\frac{\beta\gamma}{\chi}\right)\left(\frac{\varepsilon-1}{\varepsilon}\right)\left(\psi^{t-1}_t\right)^{\theta-2}\left(\psi^{*,t-1}_t\right)^{-\theta} \\ \text{and} \quad & n^*_t = \left(\frac{\varepsilon-1}{\varepsilon}\right)\left(\frac{\beta}{\chi}\right)\psi^{t-1}_t\left[(1-\theta)x_t\psi^{t-1}_t + \theta x^*_t\psi^{*,t-1}_t\right] \\ \text{and} \quad & R^*_t \geq 1 \\ \text{and} \quad & x_t \in [\beta, \bar{x}] \quad \forall t \end{aligned} \tag{36}$$

The definition of a time-consistent Home monetary equilibrium given x^* is given below.

[9] The derivation of this result is in Appendix A-2. The Nash equilibrium here is slightly different from that of Arseneau (2007) because he reverses that Cobb Douglas weights on the Foreign household consumption aggregator in order to interpret θ as country size instead of country openness as I do here.

Definition 3 (**Time-consistent Home Monetary Equilibrium given** x^*)**.** A time-consistent monetary equilibrium given the rate of Foreign money growth x^* consists of the following:

- All characteristics of Definition 1 of a private equilibrium hold.
- The sequence of Home money growth rates $\{x_t\}_{t=0}^{\infty}$ satisfy the maximization problem in (36) given the sequence of Foreign money growth rates $\{x_t^* = x^*\}_{t=0}^{\infty}$.

The next section provides a numerical example of a time-consistent Home monetary equilibrium given x^*.

3 Numerical Simulation

In this section I provide a numerical example of what the time path of a time-consistent Home monetary equilibrium looks like given different constant levels of Foreign money growth rates. This numerical exercise follows Ireland (2000). I set the openness parameter of the model in both countries to $\theta = 0.25$ implying that both countries have a 25-percent import share of GDP. The discount factor is set to $\beta = 0.95$. The elasticity of substitution is set to $\varepsilon = 6$, consistent with a 20-percent markup. The scale parameter of the disutility of labor function χ is set equal to 1. And the number of periods that households and firms use to formulate expectations is $N = 10$. The elasticities α_k in the expectations function are set to $\alpha_{10} = 1$ and $\alpha_k = 0$ for k = 1,2,...9.

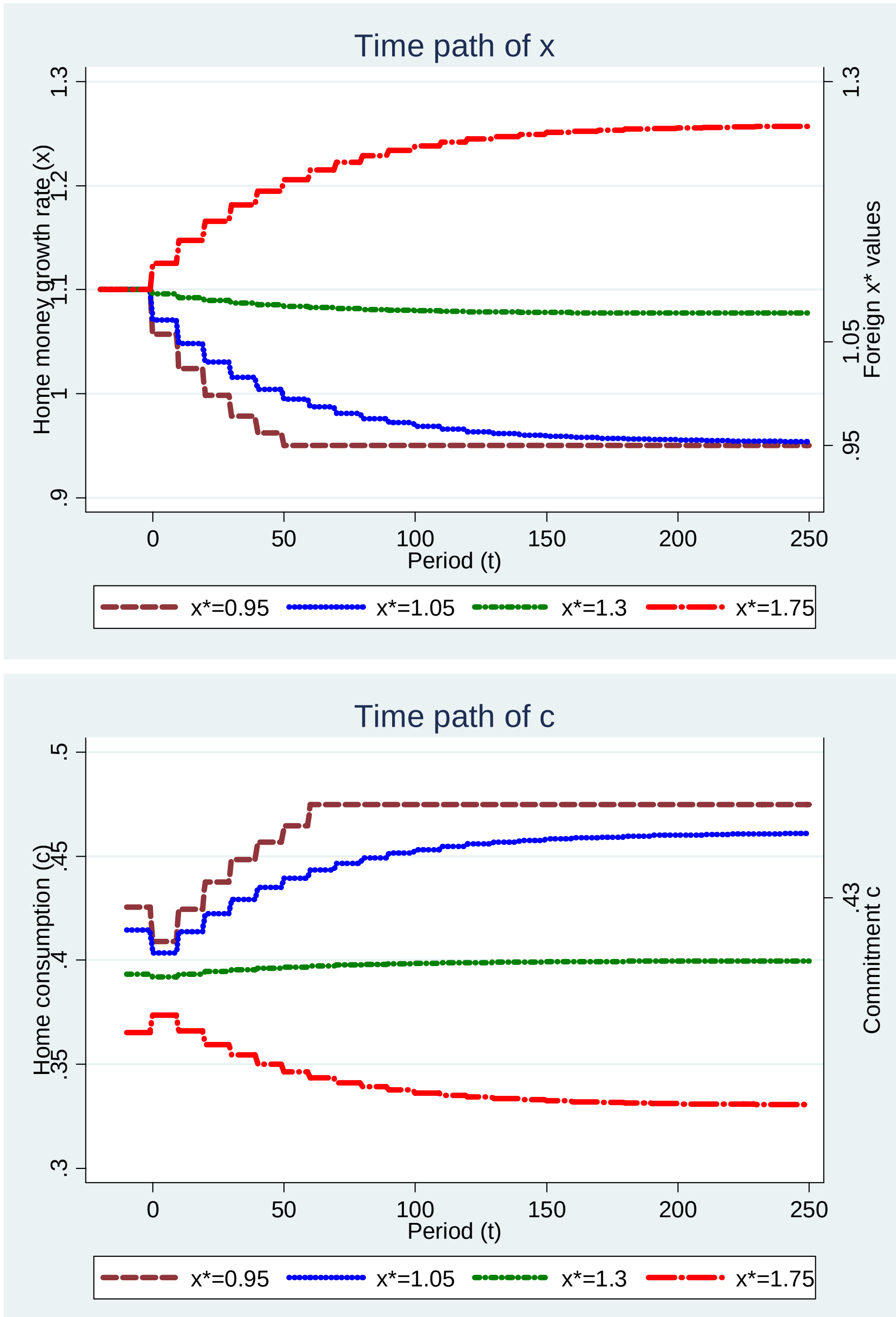
With these parameters, the optimal money growth rate with commitment that corresponds to Definition 2 and constant money growth rate rule (35) is about 1.05. The experiment here is to start the Home money growth rate at a level not equal to the monetary Nash equilibrium with commitment (1.10 in this case) and leave it there long enough that all households expect it to stay there. Then let the Home monetary authority adjust the money growth rate x_t optimally each period according to Definition 3. Figure 2 shows the time path of the Home money growth rate starting at a money growth rate of 1.1 in periods $t < 0$ but then being allowed to change its rate optimally in periods $t \geq 0$. The lower panel shows the corresponding aggregate consumption levels. Each line corresponds to a different level of constant Foreign money growth rate.

The top panel of Figure 2 shows that it is optimal for the Home monetary authority to gradually reduce the Home money growth rate for Foreign money growth rates less than 1.3. The lower panel shows how this reduction of interest rates initially inflicts a cost to societal welfare. This illustrates the key point noted by Ireland (2000) that the discount factor β is a key parameter governing the ability of a monetary authority to be able to earn credibility. That is, households must be patient enough for the long-term benefits of inflation reduction to outweigh the short-term costs. Another key characteristic of these time-consistent Home monetary equilibrium paths is that the incentive to lower inflation diminishes as the Foreign country has higher inflation.

This strategic complementarity has predicts that when a country like the United States begins a period of increased money growth rates that other central banks should follow. However, in this exercise, the Foreign central bank has to have a money growth rate of 30 percent in every period as opposed to the Home money growth rate of 10 percent. This is a wider disparity than is usually seen in major trading partners.

This experiment is merely a first pass at studying situations in which Foreign monetary policy influences Home monetary policy. An obvious next step is to construct a time-consistent Nash equilibrium. But some good findings come from this exercise. First, the steady state values in this time-consistent monetary equilibrium have more to do with the level of Foreign monetary policy than they do with the Ramsey equilibrium that comes from the commitment policy. Also, a strategic complementarity seems to exist between the Home and Foreign monetary authority. As the Foreign money growth rate increases, the incentive for the Home monetary authority to inflate goes up. Lastly, these time-consistent equilibrium fall below the upper bound of the money growth rate and do not rely on trigger strategies.

Figure 2: Time path of x for various values of x^*



Note: this graph shows the time path of the Home gross money growth rate and the corresponding aggregate Home consumption. The parameters are set to the following: $(N = 10, \chi = 1, \varepsilon = 6, \beta = 0.95, \theta = 0.25)$

4 Conclusion

The model outlined in this study provides an environment in which to study the effect of foreign monetary policy on domestic monetary policy in which the problems of multiple equilibria and dominant strategy equilibria are overcome. This result is achieved by following Ireland (2000) and relaxing the assumption of rational expectations in an intuitive way. The implications of the model are that a monetary authority can build credibility over time if households are patient enough for the long-term benefits to outweigh the short-term costs of adjusting monetary policy.

As mentioned in the previous section, an obvious extension of this work is to construct the time-consistent monetary Nash equilibrium of the two countries given different starting points. Also, the fact that the experiment of this paper starts each country at a suboptimal money growth rate begs the question of what might cause this. Adding shocks to this model that move the steady-state around would be instructive.

In addition, a large portion of the results is due to the form of the expectations functions ψ and ψ^*. One extension would be to try some more sophisticated forecasting methods that might be closer to what is seen in practice although still not as strong as the rational expectations assumption. Also, adding some imperfect information such that the Home household knows more about Home monetary policy than the Foreign household does.

APPENDIX

A-1 Derivation of Solution to Utility Maximization Problem

The maximization problem of the Home country representative household is to choose c^t_{t+j}, n^t_{t+j}, b^t_{t+j+1}, and m^t_{t+j+1} given firm prices, Home and Foreign money growth rates, and period-t information. The problem for the Foreign representative household is symmetric. The Home household maximization problem, with the constraints divided by M^S_t is the following:

$$\max \quad \sum_{j=0}^{\infty} \beta^j \left[ln(c^t_{t+j}) - \chi n^t_{t+j} \right]$$

$$\begin{aligned} \text{s.t} \quad & m^t_{t+j} + z^t_{t+j} - 1 + b^t_{t+j} - \frac{b^t_{t+j+1} z^t_{t+j}}{R^t_{t+j}} \geq p^t_{t+j} c^t_{t+j} \\ \text{and} \quad & m^t_{t+j} + z^t_{t+j} - 1 + b^t_{t+j} + w^t_{t+j} n^t_{t+j} + \int_0^1 d^t_{t+j}(h) dh \geq ... \\ & \qquad p^t_{t+j} c^t_{t+j} + \frac{b^t_{t+j+1} z^t_{t+j}}{R^t_{t+j}} + m^t_{t+j+1} z^t_{t+j} \quad \forall t \end{aligned} \tag{A.1.1}$$

Taking the derivative of the Lagrangian with respect to the choice variables and with respect to the multipliers on the constraints, the first order conditions of this problem are the following:

$$\frac{\partial \mathcal{L}}{\partial c^t_{t+j}} : \quad \frac{1}{c^t_{t+j}} = \left(\mu^t_{t+j} + \lambda^t_{t+j} \right) p^t_{t+j} \tag{A.1.2}$$

$$\frac{\partial \mathcal{L}}{\partial n^t_{t+j}} : \quad \lambda^t_{t+j} w^t_{t+j} = \chi \tag{A.1.3}$$

$$\frac{\partial \mathcal{L}}{\partial b^t_{t+j+1}} : \quad \beta \left(\mu^t_{t+j+1} + \lambda^t_{t+j+1} \right) = \left(\mu^t_{t+j} + \lambda^t_{t+j} \right) \frac{z^t_{t+j}}{R^t_{t+j}} \tag{A.1.4}$$

$$\frac{\partial \mathcal{L}}{\partial m^t_{t+j+1}} : \quad \beta \left(\mu^t_{t+j+1} + \lambda^t_{t+j+1} \right) = \lambda^t_{t+j} z^t_{t+j} \tag{A.1.5}$$

$$\frac{\partial \mathcal{L}}{\partial \mu^t_{t+j}} : \quad m^t_{t+j} + z^t_{t+j} - 1 + b^t_{t+j} - \frac{b^t_{t+j+1} z^t_{t+j}}{R^t_{t+j}} = p^t_{t+j} c^t_{t+j} \tag{A.1.6}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \lambda^t_{t+j}} : \quad & m^t_{t+j} + z^t_{t+j} - 1 + b^t_{t+j} + w^t_{t+j} n^t_{t+j} + \int_0^1 d^t_{t+j}(h) dh = ... \\ & \qquad p^t_{t+j} c^t_{t+j} + \frac{b^t_{t+j+1} z^t_{t+j}}{R^t_{t+j}} + m^t_{t+j+1} z^t_{t+j} \end{aligned} \tag{A.1.7}$$

where μ^t_{t+j} and λ^t_{t+j} are the multipliers on the cash-in-advance constraint and the budget constraint, respectively. Equations (A.1.6) and (A.1.7) hold with equality only when $\mu_{t+j}^t \geq 0$ and $\lambda_{t+j}^t \geq 0$. From (A.1.3), it is clear that $\lambda_{t+j}^t > 0$ so the budget constraint holds with equality. At this point, I assume that $\mu_{t+j}^t > 0$ so that the cash-in-advance constraint holds with equality, but this will be shown to be correct after the other variables values are derived.

If the CIA constraint holds with equality, then substituting in the market clearing conditions $m_{t+j}^t = 1$ and $b_{t+j}^t = 0$ and solving for c_{t+j}^t gives the equilibrium condition for c_{t+j}^t:

$$c_{t+j}^t = \frac{z_{t+j}^t}{p_{t+j}^t} \tag{A.1.8}$$

The expressions for $c_{t+j}^t(h)$, $c_{t+j}^t(f)$, $c_{H,t+j}^t$, and $c_{F,t+j}^t$ can be found by substituting (A.1.8) into (9) and (10). Substituting (A.1.8) into (A.1.2), solving for $\mu_{t+j}^t + \lambda_{t+j}^t$ and substituting that expression into (A.1.5) gives the following equilibrium expression for λ_{t+j}^t:

$$\lambda_{t+j}^t = \frac{\beta}{z_{t+j}^t z_{t+j+1}^t} \tag{A.1.9}$$

Substituting (A.1.9) into (A.1.3) gives:

$$w_{t+j}^t = \frac{\chi}{\beta} z_{t+j}^t z_{t+j+1}^t \tag{A.1.10}$$

Again, substituting (A.1.8) into (A.1.2), solving for $\mu_{t+j}^t + \lambda_{t+j}^t$, and substituting that expression into (A.1.4) gives the following equilibrium expression for R_{t+j}^t:

$$R_{t+j}^t = \frac{z_{t+j+1}^t}{\beta} \tag{A.1.11}$$

A-2 Miscellaneous Derivations

Derivation 1 (**Monetary Nash equilibrium with commitment**)**.** Derive the Monetary Nash equilibrium with commitment from (34) and (35) here.

References

Arseneau, D. M. (2007): "The Inflation Tax in an Open Economy with Imperfect Competition," *Review of Economic Dynamics*, 10(1), 126–47.

Barro, R. J., and D. B. Gordon (1983): "A Positive Theory of Monetary Policy in a Natural Rate Model," *Journal of Political Economy*, 91(4), 589–610.

Cooper, R. W., and H. Kempf (2003): "Commitment and the Adoption of a Common Currency," *International Economic Review*, 44(1), 119–142.

Dixit, A. K., and J. E. Stiglitz (1977): "Monopolistic Competition and Optimum Product Diversity," *The American Economic Review*, 67(3), 297–308.

Evans, R. W. (2007): "Is Openness Inflationary? Imperfect Competition and Monetary Market Power," GMPI Working Paper 3, Globalization and Monetary Policy Institute, The Federal Reserve Bank of Dallas.

Haltiwanger, J. C., and M. Waldman (1989): "Limited Rationality and Strategic Complements: The Implications for Macroeconomics," *The Quarterly Journal of Economics*, 104(3), 463–83.

Ireland, P. N. (1997): "Sustainable Monetary Policies," *Journal of Economic Dynamics and Control*, 2(1), 87–108.

——— (2000): "Expectations, Credibility, and Time-consistent Monetary Policy," *Macroeconomic Dynamics*, 4(04), 448–66.

Lucas, Jr., R. E. (1972): "Expectations and the Neutrality of Money," *Journal of Economic Theory*, 4(2), 103–24.

Romer, D. (1993): "Openness and Inflation: Theory and Evidence," *The Quarterly Journal of Economics*, 108(4), 869–903.

Wynne, M. A., and E. K. Kersting (2007): "Openness and Inflation," Staff Paper 2, Federal Reserve Bank of Dallas.

TECHNICAL APPENDIX

T-1 Derivation of Demand and Price Equations from Cost Minimization Problem

Expressions for household demand functions for individual differentiated goods $c_t(h)$ and $c_t(f)$ and within-country aggregated goods $c_{H,t}$ and $c_{F,t}$ can be derived in terms of prices and aggregate consumption c_t using the household expenditure minimization problem. This method also provides expressions for aggregate price p_t in terms of the within-country aggregate prices $p_{H,t}$ and $p_{F,t}$, as well as expressions for the within-country aggregate prices $p_{H,t}$ and $p_{F,t}$ in terms of the within-country differentiated goods prices $p_t(h)$ and $p_t(f)$. I use an individual cost-minimization problem to derive the demand functions—even though the same functions result from utility maximization—because the cost-minimization problem provides an intuitive solution for the consumer price level. The Foreign demand and price equations are derived in the same way and are symmetric.

From the lifetime utility function in equation (5), households only care about aggregate consumption as defined in (7). Furthermore, the Dixit-Stiglitz CES country-specific consumption aggregators $c_{H,t}$ and $c_{F,t}$ are defined in (6). So the household demand functions for consumption of Home-produced differentiated goods $c_t(h)$ and Foreign-produced differentiated goods $c_t(f)$ can be derived by solving the problem of the household choosing how much of each type of differentiated good to consume, given the prices of each type of good $p_t(h)$ and $p_t(f)$ and given particular levels of aggregate country-specific consumption $c_{H,t}$ and $c_{F,t}$ in order to minimize total expenditures.[10]

$$\begin{aligned} \min_{c_t(h),c_t(f)} \quad & \int_0^1 p_t(h)c_t(h)\,dh + \int_0^1 p_t(f)c_t(f)\,df \\ \text{s.t.} \quad & c_{H,t} \leq \left(\int_0^1 c_t(h)^{\frac{\varepsilon-1}{\varepsilon}}\,dh\right)^{\frac{\varepsilon}{\varepsilon-1}} \\ \text{and} \quad & c_{F,t} \leq \left(\int_0^1 c_t(f)^{\frac{\varepsilon-1}{\varepsilon}}\,df\right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad \forall t \end{aligned} \tag{T.1.1}$$

The Lagrangian for this problem is:

$$\begin{aligned} \mathcal{L} = & \int_0^1 p_t(h)c_t(h)\,dh + \int_0^1 p_t(f)c_t(f)\,df + ... \\ & p_{H,t}\left[c_{H,t} - \left(\int_0^1 c_t(h)^{\frac{\varepsilon-1}{\varepsilon}}\,dh\right)^{\frac{\varepsilon}{\varepsilon-1}}\right] + p_{F,t}\left[c_{F,t} - \left(\int_0^1 c_t(f)^{\frac{\varepsilon-1}{\varepsilon}}\,df\right)^{\frac{\varepsilon}{\varepsilon-1}}\right] \end{aligned} \tag{T.1.2}$$

[10]The dual problem of maximizing the level of aggregate consumption subject to a budget constraint of expenditures being less than the currency held at the time of exchange does not yield the same result because the multiplier on the budget constraint does not have the interpretation as the price of an extra unit of aggregate consumption.

where $p_{H,t}$ and $p_{F,t}$ are the multipliers on the two constraints and represent the marginal cost of an extra unit of aggregate country-specific consumption. So $p_{H,t}$ and $p_{F,t}$ are interpreted as the index of Home produced goods prices and the index of Foreign-produced goods prices, respectively. The first order conditions are the following:

$$p_t(h) = p_{H,t} \left(\frac{c_{H,t}}{c_t(h)} \right)^{\frac{1}{\varepsilon}} \tag{T.1.3}$$

$$p_t(f) = p_{F,t} \left(\frac{c_{F,t}}{c_t(f)} \right)^{\frac{1}{\varepsilon}} \tag{T.1.4}$$

$$c_{H,t} = \left(\int_0^1 c_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{T.1.5}$$

$$c_{F,t} = \left(\int_0^1 c_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{T.1.6}$$

Soling (T.1.3) and (T.1.4) for $c_t(h)$ and $c_t(f)$, respectively, give the following differentiated-good demand functions:

$$c_t(h) = \left(\frac{p_t(h)}{p_{H,t}} \right)^{-\varepsilon} c_{H,t} \tag{T.1.7}$$

$$c_t(f) = \left(\frac{p_t(f)}{p_{F,t}} \right)^{-\varepsilon} c_{F,t} \tag{T.1.8}$$

Substituting (T.1.7) and (T.1.8) back into (T.1.5) and (T.1.6) gives the following two expressions for $p_{H,t}$ and $p_{F,t}$ in terms of $p_t(h)$ and $p_t(f)$:

$$p_{H,t} = \left(\int_0^1 p_t(h)^{1-\varepsilon} dh \right)^{\frac{1}{1-\varepsilon}} \tag{T.1.9}$$

$$p_{F,t} = \left(\int_0^1 p_t(f)^{1-\varepsilon} df \right)^{\frac{1}{1-\varepsilon}} \tag{T.1.10}$$

Likewise, the household demand functions for consumption of Home-produced good $c_{H,t}$ and Foreign-produced good $c_{F,t}$ can be derived by solving the problem of the household choosing how much of each type of country-specific aggregate good to consume, given the prices of each type of good $p_{H,t}$ and $p_{F,t}$ and given a particular level of aggregate consumption c_t, in order to minimize total expenditures.

$$\min_{c_{H,t}, c_{F,t}} \; p_{H,t} c_{H,t} + p_{F,t} c_{F,t} \quad \text{s.t.} \quad c_t \leq (c_{H,t})^{1-\theta} (c_{F,t})^{\theta} \quad \forall t \tag{T.1.11}$$

The Lagrangian for this problem is:

$$\mathcal{L} = p_{H,t} c_{H,t} + p_{H,t} c_{H,t} + p_t \left[c_t - (c_{H,t})^{1-\theta} (c_{F,t})^{\theta} \right] \tag{T.1.12}$$

where p_t is the multiplier on the constraint and represents the marginal cost of an extra unit of aggregate consumption. So P_t is interpreted as the price of aggregate consumption. The first order conditions are the following:

$$p_{H,t} = (1-\theta)p_t \left(\frac{c_{F,t}}{c_{H,t}}\right)^{\theta} \tag{T.1.13}$$

$$p_{F,t} = \theta p_t \left(\frac{c_{H,t}}{c_{F,t}}\right)^{1-\theta} \tag{T.1.14}$$

$$c_t = (c_{H,t})^{1-\theta} (c_{F,t})^{\theta} \tag{T.1.15}$$

Dividing (T.1.13) by (T.1.14) gives the following relationship:

$$\frac{p_{H,t}c_{H,t}}{p_{F,t}c_{F,t}} = \frac{1-\theta}{\theta} \tag{T.1.16}$$

Solving (T.1.13) and (T.1.14) for $c_{H,t}$ and $c_{F,t}$, respectively, and substituting in (T.1.15) gives Home demand equations for aggregate consumption of Home goods and aggregate consumption of Foreign goods.[11]

$$c_{H,t} = (1-\theta)\left(\frac{p_{H,t}}{p_t}\right)^{-1} c_t \tag{T.1.17}$$

$$c_{F,t} = \theta \left(\frac{p_{F,t}}{p_t}\right)^{-1} c_t \tag{T.1.18}$$

These demand equations are analogous to the differentiated-good demand equations in (T.1.7) and (T.1.8), except that they include the home-bias parameter. Substituting (T.1.17) and (T.1.18) back into (T.1.15) gives the expression of the consumer price index in the Home country p_t in terms of $p_{H,t}$ and $p_{F,t}$.

$$p_t = \frac{1}{\gamma}(p_{H,t})^{1-\theta}(p_{F,t})^{\theta} \quad \text{where} \quad \gamma = (1-\theta)^{1-\theta}\theta^{\theta} \tag{T.1.19}$$

Lastly, $c_t(h)$ and $c_t(f)$ can be expressed in terms of prices and c_t by substituting (T.1.17) and (T.1.18) into (T.1.7) and (T.1.8), which results in:

$$c_t(h) = (1-\theta)\left(\frac{p_t(h)}{p_{H,t}}\right)^{-\varepsilon}\left(\frac{p_{H,t}}{p_t}\right)^{-1} c_t \tag{T.1.20}$$

$$c_t(f) = \theta\left(\frac{p_t(f)}{p_{F,t}}\right)^{-\varepsilon}\left(\frac{p_{F,t}}{p_t}\right)^{-1} c_t \tag{T.1.21}$$

[11]Equations (T.1.16), (T.1.17), and (T.1.17) show that expenditures on imports are a constant fraction of GDP, represented by θ. This is the foundation for using θ as representing a country's degree of openness to international trade.

T-2 Comments

- figure out the transversality condition that is given in Ireland (2000) and Arseneau (2007).

- Do the model for $\sigma \neq 1$. When $\sigma \in (0,1)$, solving for the prices requires the solution to a multi-equation system of first-order difference equations. However, when $\sigma > 1$ the solution to the difference equations may blow up. It might require me putting some other requirements on the system in order for it not to blow up.

- Extensions to work on

  - Do stochastic setup so the unique steady state moves around. This would put the economy always in transition.

    - Define output gap as the deviation of current output from its steady state value.

  - Some imperfect information or learning might be good. Volker Wieland in Germany has some good stuff on dynamic programming with learning.

  - Do different forms of ψ_1, ψ_2, ψ_1^*, and ψ_2^* including forecasting techniques and maybe even Taylor Rules.

  - Look at the case where $z_{t+j}^{t-1,h} \neq z_{t+j}^{t-1,f}$ and where $z_{t+j}^{*,t-1,h} \neq z_{t+j}^{*,t-1,f}$. Agents in each country have different expectations and/or information about the money growth rate in the other country.

  - Ireland (2000) shows that the assumption of rational expectations is the key characteristic generating the multiplicity of reputational equilibria and the autarkic nonreputational equilibria in the more fully specified version of the time-consistent monetary policy problem. This same difficulty exists in a two-country version of Ireland (2000). However, Haltiwanger and Waldman (1989) points to models with strategic complementarities as the source of the multiplicity of Pareto ranked equilibria. He shows that a model in which some agents are "sophisticated" and have forward-looking rational expectations and in which some agents are "naive" and have backward-looking adaptive expectations provides very different results from the standard rational expectations models. Cooper suggested that a good exercise would be to say that a portion of the agents λ have backward-looking adaptive expectations and the rest of the agents $1-\lambda$ have forward-looking rational expectations. Ireland (2000) shows that when all agents have adaptive expectations $\lambda = 1$ that a unique steady state exists. An interesting question would be to find out what the critical value is λ^* at which a multiplicity of equilibria can be found. This could also be a good framework for looking at the effects of emerging economies opening up to trade and financial interaction with more developed economies. The idea is that emerging economies have a larger proportion of naive agents.

- Why I ditched state-dependent pricing
  - The model doesn't work with rational expectations time-consistent equilibrium
    - $z_{t+j}^{t-1} = x_{t+j}$ so there is never any reason for firms or individuals to change price.
  - Could have an effect in the Ireland (2000) version with adaptive expectations, but all it does is change the speed of transition between steady states. Not very interesting.
  - State-dependent pricing is best when monetary authority is stochastic. It might be good if there was imperfect information or the stochastic version of the model where the steady state is jumping around.
-